

A Model for Turbulent Flame Speed Based on Theory of Turbulent Flame Brush Thickness Dynamics

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ABSTRACT

In this work, a new theory is proposed for determining the instantaneous turbulent flame speed using a simplified but physics-based approximation of the dynamics of the transient turbulent flame brush thickness. A summary of the main points of this new model's derivation are reviewed, which can be thought of as an extension of Damköhler's steady state flame speed model to include the effect of the transient development of a turbulent flame. The resulting flame brush thickness dynamics model is compared against historical measurements available in the literature as well as against 3D realizations of resolved turbulent flame propagation using the instantaneous G-equation. This comparison shows the newly derived model can describe both the transient and steady state turbulent flame speed across a range of conditions with a singular modeling form and is therefore expected to find utility as turbulent flame speed model for combustion devices where representation of the developing turbulent flame is of crucial importance.

1. Motivation and Premise of the Present Analysis

Replacement of the chemical source terms in the energy equation with an alternative flame kinematic term is a convenient simplification that is commonly applied in turbulent combustion modeling. This has given rise to the concept of a turbulent flame speed and for decades researchers have proposed numerous models to characterize the propagation rate of the wrinkled turbulent flame.

The common approach for modeling the instantaneous turbulent flame speed, s_T , simplifies the problem by separating the steady state flame speed, $s_{T,\infty}$, from the transient response of the flame speed shown by the right-hand side (RHS) of Eq. (2).

$$\frac{s_{T,\infty}}{s_L} = 1 + \frac{\bar{u}'}{s_L} \quad (1)$$

$$\frac{s_T}{s_{T,\infty}} = 1 - \exp(-\tau/t) \quad (2)$$

There are several examples of steady state turbulent flame speed models available in the literature with a similar form to Eq. (1). A selection of these models⁽¹⁾⁻⁽¹⁵⁾ have been plotted in Fig. 1. Note that despite having the same input

conditions⁽¹⁶⁾, the range in predicted values of the turbulent flame speed ratio, s_T/s_L , at a given relative turbulence intensity, $\bar{u}'/s_L$, spans across an order of magnitude.

This variation in modeled turbulent flame speed ratios is

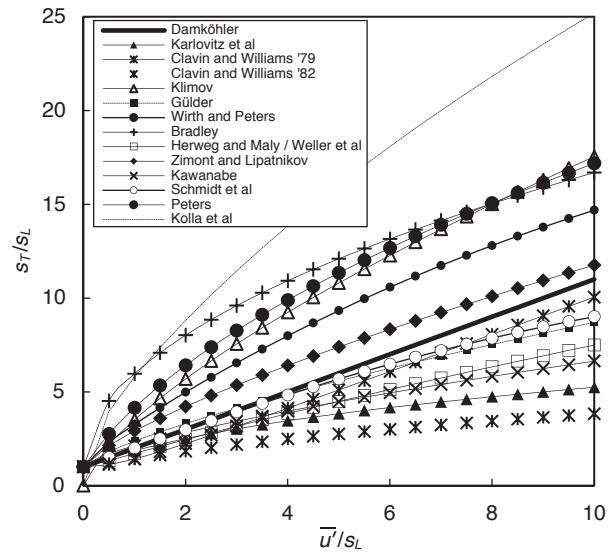


Fig. 1 Comparison of steady state flame speed ratios predicted by various models at typical engine conditions

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reflective of the range of the observed turbulent flame speed ratios in the literature. Turbulent flame speed measurements compiled by Abdel-Gayed⁽¹⁷⁾⁻⁽²³⁾ from all known turbulent flame speed measurements at the time are shown in Fig. 2. Note the similar span of values in measured turbulent flame speed ratios compared to the predicted values presented in Fig. 1. Indeed, many of these models referred to this same measurement database during their development.

Models of the turbulent flame speed shown in Fig. 2 tend to be empirical fits rather than phenomenological or physics-based models. This empiricism in the turbulent flame speed models will lead to necessary modeling calibration of the turbulent flame speeds during their application. Ultimately this limits the ability of many flame kinematics-based approaches to represent turbulent combustion processes rather than simply representing the details of a specific dataset used to train a particular model. As it is desirable to minimize this type of empiricism while increasing representation of the relevant turbulent flame propagation phenomena, the primary focus of this work was to develop a simple, physics-based model capable of capturing the macroscopic dynamic phenomena of turbulent flame propagation.

An alternative to the typical empirical fitting functions encountered for the steady and transient development of s_T , like Eqs. (1) and (2), might be to identify the underlying influential terms that lead to the dynamic response of the flame speed and then to allow the steady state condition to be developed naturally once dynamic equilibrium has been reached. A model of transient turbulent flame speed based on this principle of dynamic equilibrium does not appear to have been widely explored throughout the literature. This concept will be the main approach investigated here.

The remaining structure of this paper is as follows:

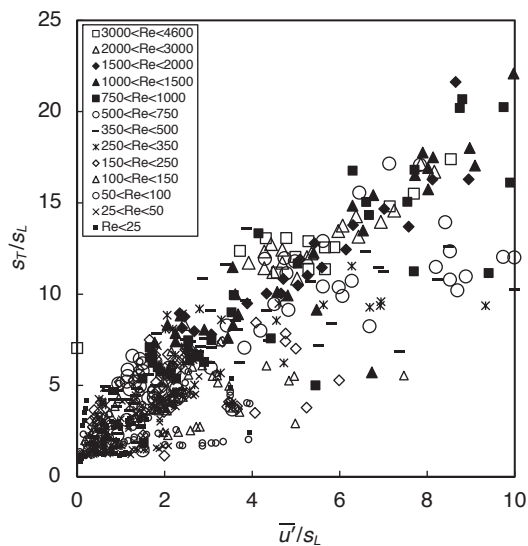


Fig. 2 Steady state turbulent flame speed measurements compiled from many sources by Abdel-Gayed

Chapters 2-4 discusses a detailed analysis of resolved 3D turbulent flame propagation using the G-equation, which was used to better understand the underlying mechanics of turbulent flame propagation. The results of this analysis lead to the development of a new Turbulent Flame Brush Thickness Dynamics (FBTD) model which is derived in Chapter 5. The FBTD model results are first compared against the measurement results of Abdel-Gayed in Chapter 6, and then against realized flame propagation of the instantaneous 3D G-equation through a synthetic frozen turbulent flow field in Chapter 7. Finally, a summary of the key findings and discussion of the results is given in Chapter 8. Readers should note that the present work is a summary of the main findings from a more comprehensive investigation by the author which is available elsewhere in the literature⁽¹⁶⁾.

2. The G-equation

A simple yet fundamental model for studying the details of a developing turbulent flame is the instantaneous G-equation of Williams⁽²⁴⁾,

$$\frac{\partial G}{\partial t} + \frac{\partial G}{\partial x_f} \vec{u} = s_L \left| \frac{\partial G}{\partial x_f} \right| \quad (3)$$

The G-equation is a member of the level set approach⁽²⁵⁾,⁽²⁶⁾ where a self-propagating interface interacts with an embedded velocity field. If the instantaneous velocity vector, $\vec{u}$, can be attributed to the fluctuating velocity of turbulence, and if the local normal propagation speed of the G_0 interface can be attributed to the laminar flame speed, s_L , then the resulting motion of the G level set at G_0 can be interpreted as an instantaneous realization of a turbulent flame, and so is suggested to be a suitable model for the purposes of the present analysis.

The instantaneous G-equation will be simulated for a set of conditions that are relevant for practical devices. The results will then be studied with the intent of ascertaining, and then modeling, the underlying influential behaviors most responsible for the unsteady and dynamic behavior of turbulent flame propagation.

The use of the G-equation for this purpose requires several assumptions to be made. The effect of turbulent motions on the flame are implicitly assumed to be entirely kinematic in nature, which is consistent with the flamelet regime of combustion⁽²⁷⁾. Equivalently, this assumption implies the chemistry is infinitely fast and therefore the flame surface is infinitely thin, both suitable approximations for typical premixed engine combustion. The use of William's form of the G-equation does not include stretch corrections to the laminar flame speed, and this assumption is less justifiable, as even weakly turbulent flames have regions of the flame surface which experience significant curvature effects⁽²⁸⁾. However, such impacts are presumed

to be highly local and short-lived phenomena, and so it has been decided to neglect these impacts in the aim of simplicity. Additionally, inclusion of the isothermal assumption makes the solution of the G-equation trivial, as the flame propagation becomes decoupled from the flow, and was already decoupled from the chemistry by the flamelet assumption. Research has indicated that the expansion ratio across the flame have only a very weak dependence on the turbulent flame speed⁽²⁹⁾, and so once again this assumption will be kept in the aim of simplicity.

The input laminar flame speed can be defined arbitrarily as it represents an external source term. However, a realistic instantaneous turbulence velocity field is required to continue the analysis, and this cannot be defined arbitrarily. In prior works the fluctuation velocity is frequently precomputed⁽³⁰⁾ using a grid structure or other means of inducing turbulent flow. In some examples a solenoidal fluctuating velocity is specified using a Gaussian random variable which conforms with a chosen energy spectrum⁽³¹⁾. In either case, coupling of the flow solver with an intense swirling flow is known to rapidly dissipate the fluctuating velocity unless a source of energy is used to sustain the fluctuations. To help prevent this, forcing functions have been used to artificially maintain the fluctuating velocity by reintroducing energy at a desired filter length⁽³²⁾, where typically this length is associated with the integral length scale of turbulence.

In this work a different approach will be used to mitigate the effects of turbulence energy dissipation. The flow field will be treated as frozen and non-interacting with the flame propagation, which when combined with the assumptions discussed above greatly reduces the numerical burden of the analysis while still providing insight into the fundamental transient behavior of a developing turbulent flame.

The details of this method will be discussed in the following chapter.

3. Synthetic Turbulent Flow Field

The flow field will be synthesized using the following scheme which was devised to generate a realistic turbulent flow field with a high degree of control during initiation without requiring a detailed solution of the associated conservation equations. First the desired length scale of turbulence, l_0 , is passed as an input which sets the overall scale of the problem to be solved. The size of the box-shaped computational domain is then specified according to the number of eddy lengths desired in each of the three Cartesian coordinates; N_i, N_j, N_k . The total volume of the computational domain, V_{cd} , can be estimated by,

$$V_{cd} = N_i N_j N_k l_0^3 \quad (4)$$

If an individual eddy can be assumed to have a local circular cross-sectional area, and if it spans the domain with a length $V_{cd}^{1/3}$, then the expected volume of an individual

eddy can be estimated as,

$$V_{eddy} = \frac{\pi}{4} l_0^2 V_{cd}^{1/3} \quad (5)$$

This leads to an estimate for the total number of eddies required to fill the computational domain to the desired span as,

$$N_{eddy} = V_{cd} / V_{eddy} = V_{cd}^{2/3} / \left(\frac{\pi l_0^2}{4} \right) \quad (6)$$

Substituting (4) into (6) provides the total number of eddies to be initialized in terms of the eddy lengths desired in the three Cartesian coordinates.

$$N_{eddy} = \frac{4}{\pi} (N_i N_j N_k)^{2/3} \quad (7)$$

To initialize the velocity field the location of each individual eddy within the computational domain should be anchored in space. These datum points are positioned within the subsections of the entire domain using randomly assigned coordinates, with the intention of randomly, but evenly, distributing the eddies. Each eddy's 3D centerline is specified in the form of a randomly determined 2nd order parametric polynomial of the form,

$$L = L_i \hat{i} + L_j \hat{j} + L_k \hat{k} \quad (8)$$

$$L_n(t) = \alpha_n t^2 + \beta_n t + \gamma_n \quad (9)$$

Where α, β, γ are randomly determined coefficients for each of the n dimensions (i, j , and k) of the centerline specified for each eddy along the parametric dimension t .

The 3D velocity field of an individual eddy is then determined by using the normal to the vortex center line by use of a proposed variant of the solenoidal 2D Lamb-Oseen vortex. The tangential velocity of the Lamb-Oseen vortex is given by,

$$u_\theta = \frac{\Gamma}{2\pi r} g(r, t) \quad (10)$$

Where the first term represents the tangential velocity profile of an irrotational vortex from 2D potential flow theory, and the function $g(r, t)$ represents the viscous core of a rotational vortex given by,

$$g(r, t) = 1 - \exp(-r^2/4\nu t) \quad (11)$$

The Lamb-Oseen vortex is intended to model a decaying line vortex over time due to viscosity, but it can be modified to allow the specification of a rotational vortex core of diameter, l_0 , by replacing the function $g(r, t)$ with the following,

$$\gamma(r, t) = 1 - \exp\left(-\frac{1}{2}\left(\frac{r\pi}{l_0}\right)^2\right) \quad (12)$$

The variable r is the radial-normal distance to the vortex centerline. Example plots of the modified Lamb-Oseen vortex described by Eqs. (10) and (12) are shown in the Fig. 3 below. It is assumed that the rotational vortex core which contains the vorticity and rotational inertial of the turbulent eddies is a likely candidate feature of the vortex to be associated with the integral length scale of turbulence as a convenience. As justification, it can be seen than the radial distance between the peak tangential velocities across the diameter of the eddy is found to be near the specified

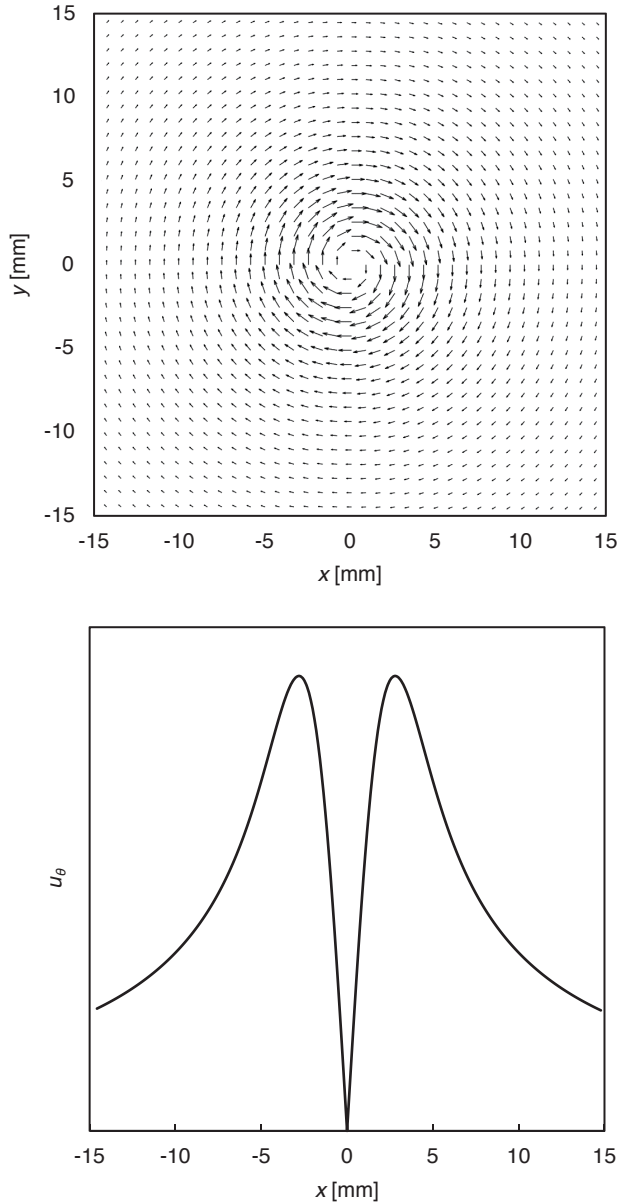


Fig. 3 Example 2D vortex velocity vectors and tangential centerline velocity from Eqs. (10)-(12)

integral length scale of 3 mm.

Next the radial distance r will be determined for every cell center in the computational domain. In a 2D profile this can simply be found as the radial distance within the plane to the specified vortex center point. In 3D however, the radial distance r of an arbitrary point is defined as the shortest distance from that point to the vortex centerline. This shortest distance can be found by solving the following optimization problem,

$$t_{min} \rightarrow \min(\text{dist}\{p, L(t)\}) \quad (13)$$

The minimum distance found with the equation above is associated with the optimal parametric value of $t_{min}(i, j, k)$ which is unique for each point $p(i, j, k)$ in the domain for every individual eddy. Point p has coordinates p_i, p_j, p_k , and point $L(t)$ has components L_i, L_j, L_k at the parametric line distance t , each defined in Euclidean space. Having determined these geometric points within the domain, the radial distance r can then be found by application of the Pythagorean theorem by,

$$r = \left[(L_i(t_{min}) - p_i)^2 + (L_j(t_{min}) - p_j)^2 + (L_k(t_{min}) - p_k)^2 \right]^{1/2} \quad (14)$$

The figure below shows the graphical relationship between the parametric curve, $L(t)$, the arbitrary point within the domain, p , the closest point on the curve L to point p shown at $L(t_{min}, p)$, and finally, the distance r between these two points.

The total velocity field is then found by superimposing the individual contribution from N_{eddy} number of eddies within the volume V_{cd} . In Fig. 5 below, the isosurfaces of velocity magnitude after the frozen turbulent flow field has been fully synthesized are shown. As a qualitative comparison, an image of DNS isotropic homogenous box

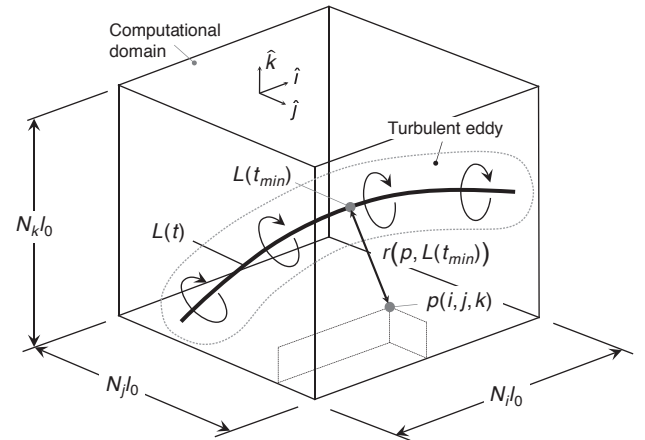


Fig. 4 Graphical representation of the distance r between points p and L at t_{min}

turbulence simulations of Chang et al⁽³³⁾ are presented in Fig. 6 using the Q-criterion, which after review suggests that the artificial flow field synthesized is reasonable.

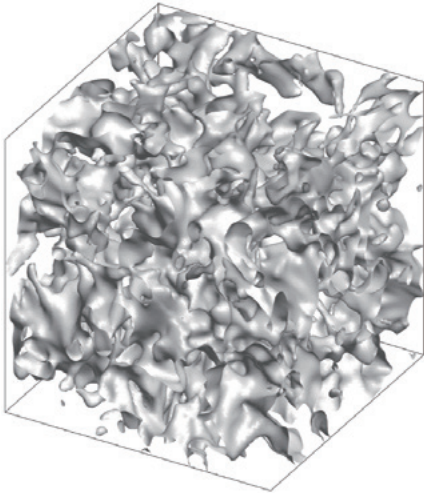


Fig. 5 Synthetic turbulent flow field initiation with 150 eddies inside the computational domain

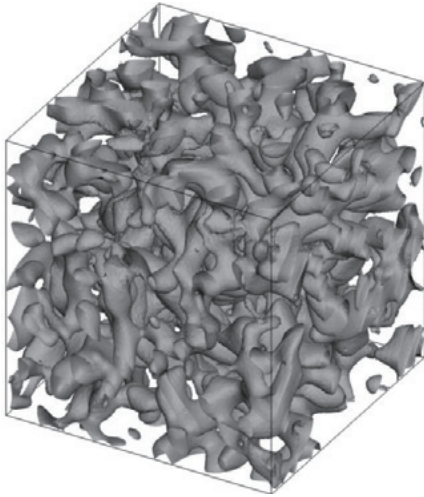


Fig. 6 DNS results of an isotropic turbulent flow by Chang et al

4. The G-equation Results

Following from the scheme described in Chapter 4, the box shaped computational domain was initiated using the following values for the relevant parameters: $l_0 = 2$ mm, $N_i = 20$, $N_j = 20$, $N_k = 40$, $N_{eddy} = 809$, $V_{cd} = 128,000$ mm³, #cells = $120 \times 120 \times 240$.

Shown in Fig. 7, the G field was initiated to give an initially planar flame surface located just inside the domain to avoid the possibility of eddy velocity advecting

a portion of the flame surface outside of the computational domain where it can no longer be tracked. The G_0 level set value was chosen arbitrarily as 0.5, and G upstream of this value was initialized to 0, while downstream values were initialized to 1. Cyclic boundary conditions were implemented to walls normal to the $\hat{i}$ and $\hat{j}$ directions and fixed values of G were applied to the walls normal to the $\hat{k}$ direction.

Figure 8 shows nine sequential timesteps of the G-equation solution for $\bar{u}'/s_L = 1$. The initially planar surface undergoes a significant transient period while the

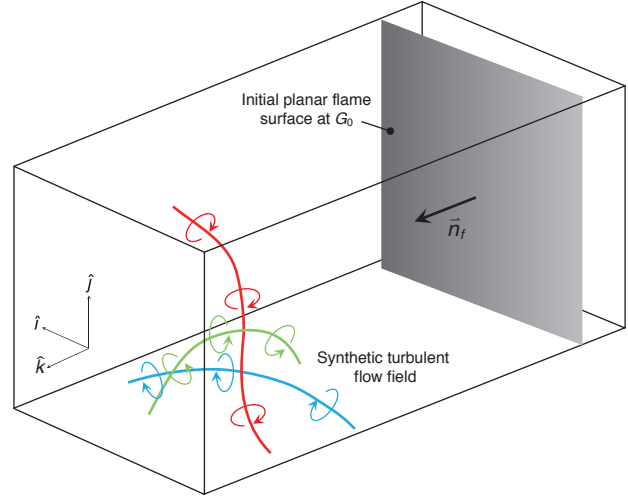


Fig. 7 Depiction of the initially planar flame surface at G_0 propagating into a synthetic frozen turbulent flow field

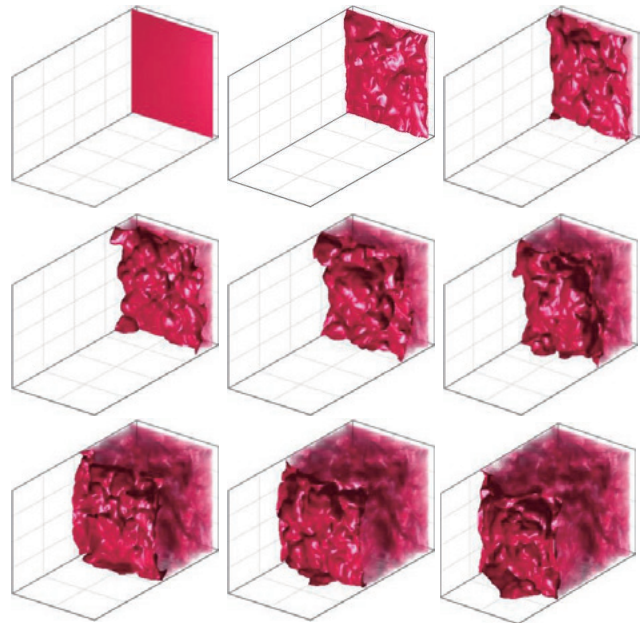


Fig. 8 Example G-equation results of the transient development of an initially planar flame unity turbulence intensity

turbulent flow begins to wrinkle and distort the surface before ultimately reaching a quasi-steady propagation rate.

After careful study of these representations of turbulent flame propagation like shown in Fig. 8, it became apparent to the author that the dynamics of the flame brush thickness might be usable as a simpler means of predicting the turbulent flame speed.

Traditionally, the mechanism for the increased consumption rate of turbulent flames has been associated with the increased wrinkled flame area, A_L . This is shown explicitly in modeling equalities like,

$$\frac{dm_b}{dt} = \rho_u A_L S_L = \rho_u A_T S_T \quad (15)$$

Which lead to,

$$s_T = \frac{A_L}{A_T} s_L \quad (16)$$

Where for a fixed laminar flame speed, Eq. (16) shows that increasing the wrinkled laminar flame surface area A_L will result in a proportionally faster apparent turbulent flame speed, s_T . These conclusions are not debated here. As depicted in Fig. 9, increasing turbulence intensity will increase the wrinkling of the flame, but additionally it can be seen that on average more of the flame is not

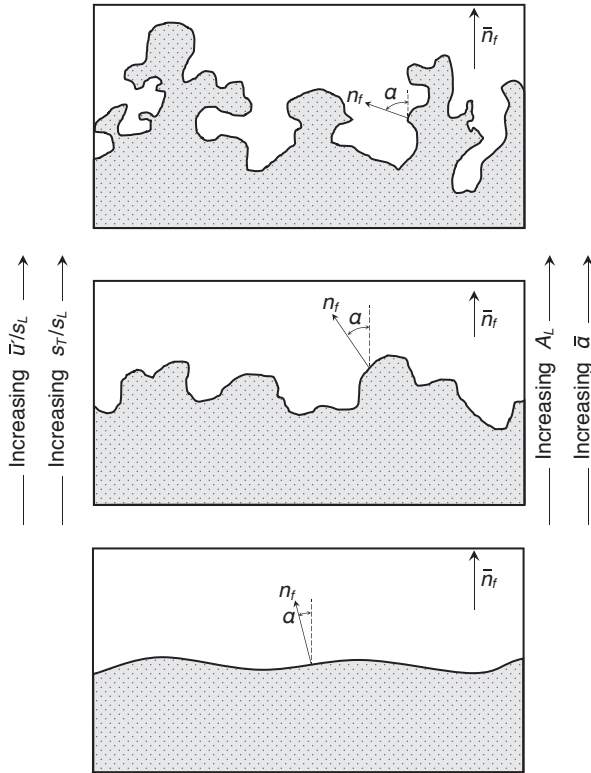


Fig. 9 Depiction of influence to various turbulent flame properties as turbulence intensity is increased

locally propagating along the mean vector, $\bar{n}_f$, but instead propagating locally along n_f at some angle α with respect to that mean propagation vector.

The statements above are not a new observation, and in fact many turbulent flame speed models already implicitly include this influence of a deviating flame angle with increasing turbulence intensity. For example, this is essentially the same concept that led to Damköhler's original turbulent flame speed model⁽¹⁾ shown here,

$$\frac{s_{T_{\infty}}}{s_L} = 1 + \frac{\bar{u}'}{s_L} \quad (17)$$

Where it was assumed the mechanism for enhanced flame speed was due to a proportional increase in the flame angle α and not strictly due to the increasing flame area itself. However, Damköhler's model only intended to predict the steady state turbulent flame speed and does not consider the dynamic relationship between these flame variables. These kinds of dynamic interactions will now be captured by introducing a new theory of turbulent flame brush thickness dynamics.

5. Turbulent Flame Brush Thickness Dynamics

An approximation for the mean flame angles $\bar{\alpha}$ and $\bar{\theta}$ can be found from the stencil in Fig. 10 below. Here the same characteristic triangle is used that Damköhler applied, but the characteristic lengths of the sides of the triangle have been assigned to the flame brush thickness and the integral length scale of turbulence, rather than the laminar flame area and the turbulent flame area.

By referencing the geometry in the figure below, it can be seen that an approximation for the mean flame angle, $\bar{\theta}$, can be found by,

$$\tan \bar{\theta} = l_{fb}/l_0 \quad (18)$$

For a very thin flame brush ($l_{fb} \rightarrow 0$) the angle $\bar{\theta}$ approaches zero radians, and for a very thick flame brush ($l_{fb} \rightarrow \infty$), the angle $\bar{\theta}$ approaches $\pi/2$ radians. The angle $\bar{\theta}$

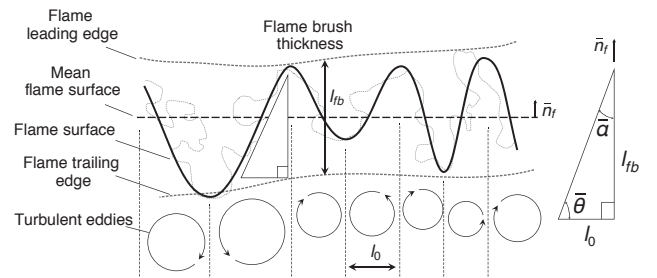


Fig. 10 Turbulent flame angle relationship by mean turbulence and turbulent flame geometrical length scales

can be thought of as the mean local deviation angle from the global mean propagation vector. Therefore, when considering a purely laminar flame that has no flame brush thickness, $\bar{\theta}$ is zero. This indicates that locally the flame is propagating in the mean normal vector everywhere, and that there is no deviation from the mean, as would be expected. While a flame brush approaching infinite thickness would yield $\bar{\theta}$ that is approaching $\pi/2$ radians. Indicating that an infinitely thickened flame brush is propagating perpendicularly to the mean on average.

5.1. Flame Brush Thickness Growth

Figure 11 shows a depiction of an initially zero thickness flame brush being thickened by a field of nearly homogeneous and isotropic turbulence across a span of finite time Δt . Note that while isotropic turbulence has no statistical directional preference, it is the aligned eddies that have the greatest contribution to the instantaneous flame brush thickness growth. If a turbulent eddy is assumed to be a vortex or swirling region of flow where a local axis of rotation can be defined, then it is the eddies whose rotation axis are aligned perpendicularly to the mean propagation vector, $\bar{n}_f$, which have the highest effective brush thickness growth. This has been shown explicitly in the 2D depiction of the flame brush growth process below of Fig. 11.

Here the common counter rotating vortex structure has been assumed. This implies that aligned eddies can both advance and retreat the front as the flame passes through the counter rotating vortex pair. Using this stencil an estimate for the discrete change in the flame brush thickness due only to the effect of turbulence, and ignoring all other factors, is given by,

$$\left. \frac{\Delta l_{fb}}{\Delta t} \right|_{growth} = \Delta t \{ (s_L + u_{ch}) - (s_L - u_{ch}) \} = 2u_{ch}\Delta t \quad (19)$$

Where u_{ch} is a characteristic turbulence velocity associated with the maximum and minimum instantaneous velocities normal to $\bar{n}_f$. Due to local rotational symmetry

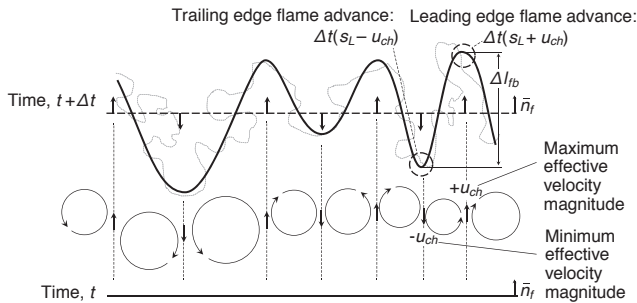


Fig. 11 Stencil of an initially planar flame at time t being thickened by a field of eddies after a discrete increment of time

of turbulent eddies, the maximum and minimum values are assumed to be equal in magnitude but opposite in sign.

The characteristic velocity u_{ch} is an inherently instantaneous quantity, but it can be estimated from the statistical $\bar{u}'$ by assuming that the rms fluctuation velocity is a Gaussian distributed random variable. Higher order statistics of small scale turbulence are known to be non-Gaussian⁽³⁴⁾, but the statistics of the large scale turbulence, especially near the core of the probability density function, are often assumed to be quite well approximated by a Gaussian distribution⁽³⁵⁾. As it is explicitly assumed that the energy containing eddies are the ones most responsible for thickening of the flame brush, a suitable estimate for the maximum instantaneous fluctuation velocity experienced by the flame over the interval Δt can be found from,

$$u'_{max} = u'_{min} \approx 3\bar{u}' \quad (20)$$

Where it has been assumed that six standard deviations cover the statistical range of the fluctuation velocity for both the maximum and minimum instantaneous fluctuation velocity.

Following along the approximate flame streamlines (the dotted vertical lines in Fig. 11), it is assumed that the instantaneous fluctuation velocity experienced by the flame varies between u'_{max} and u'_{min} and follows in a nearly sinusoidal path according to the wave length $2l_0$. This allows the average characteristic velocity experienced by the flame brush, u_{ch} , to be estimated by finding the average positive values of the sine wave over half of one cyclic period,

$$u_{ch} \approx u'_{max} \frac{1}{\pi} \int_0^\pi \sin \phi d\phi \quad (21)$$

The mean characteristic velocity of the turbulent fluctuations is then found to be,

$$u_{ch} \approx \frac{6}{\pi} \bar{u}' \quad (22)$$

Substituting Eq. (22) into (19) and taking the limit as $\Delta t \rightarrow 0$ we find the following differential modeling approximation for the flame brush thickness growth term as a function of the rms fluctuation velocity.

$$\lim_{\Delta t \rightarrow 0} \left. \frac{\Delta l_{fb}}{\Delta t} \right|_{growth} = \left. \frac{dl_{fb}}{dt} \right|_{growth} = \frac{12}{\pi} \bar{u}' \quad (23)$$

5.2. Flame Brush Thickness Decay

In the absence of other effects, a thickened flame brush will tend to thin over time due to self-annihilation caused by regions of the flame surface intersecting with one another. This effect is also commonly referred to as the kinematic

restoration^{(36), (37)} of the flame surface area. An estimate for the discrete quantity of the flame brush thickness that decayed over the time interval Δt can be determined from the stencil shown in Fig. 12. In this image a segment of the thickened flame brush from Fig. 11 has been depicted as if in isolation from the turbulence.

In the image below it can be seen that the trailing edge of a wrinkled self-normal propagating surface has an apparently faster rate of travel as compared to the leading edge. This is due to the self-annihilation mechanism which is a function of the mean local deviation angle, $\bar{\theta}$. An estimate for the discrete change in the flame brush thickness due only to the effect of self-annihilation and ignoring all other factors is therefore given by,

$$\begin{aligned} \left. \frac{\Delta l_{fb}}{\Delta t} \right|_{decay} &= \Delta t \{ (s_L) - (s_L / \cos \bar{\theta}) \} \\ &= s_L (1 - \sec \bar{\theta}) \Delta t \end{aligned} \quad (24)$$

As before with the brush thickness growth term, the time derivative for the brush thickness decay due to self-annihilation is found by rearranging Eq. (24) and taking the limit as $\Delta t \rightarrow 0$.

$$\begin{aligned} \lim_{\Delta t \rightarrow 0} \left. \frac{\Delta l_{fb}}{\Delta t} \right|_{decay} &= \left. \frac{dl_{fb}}{dt} \right|_{decay} \\ &= s_L (1 - \sec \bar{\theta}) \end{aligned} \quad (25)$$

5.3. Freely Propagating Turbulent Flames

A dynamic equation for describing a freely propagating turbulent flame brush is given below, where freely propagating in this context specifically excludes the thickness destruction due to flame-wall interaction. These effects will be considered later in Section 5.5. Here a

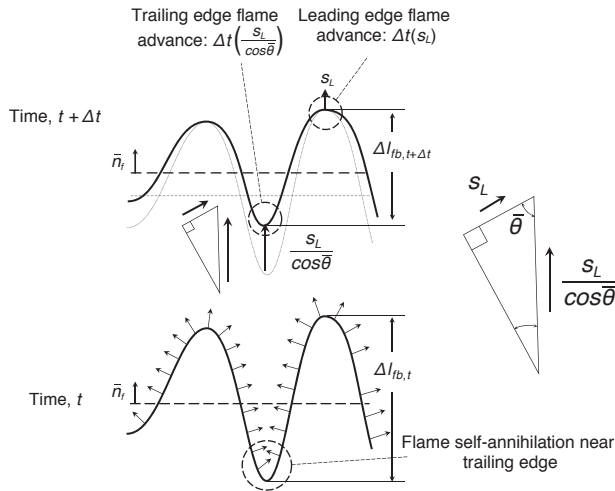


Fig. 12 Depiction of a thickened flame brush at time t thinning by self-annihilation after discrete time increment Δt

freely propagating turbulent flame brush is assumed to be influenced only by thickness growth due to turbulent stretching and thickness decay due to self-annihilation.

$$\left. \frac{dl_{fb}}{dt} \right|_f = \left. \frac{dl_{fb}}{dt} \right|_{growth} + \left. \frac{dl_{fb}}{dt} \right|_{decay} \quad (26)$$

Substituting in terms defined in Sections 5.1. and 5.2. yields the following,

$$\left. \frac{dl_{fb}}{dt} \right|_f = \frac{12}{\pi} \bar{u}' + s_L (1 - \sec \bar{\theta}) \quad (27)$$

Note that $\bar{\theta}$ is a nonlinear function of the flame brush thickness from the definition below,

$$\bar{\theta} = \tan^{-1}(l_{fb}/l_0) \quad (28)$$

It is seen that the derived equation for the freely propagating turbulent flame brush thickness dynamics, Eq. (27), is a first order nonlinear ordinary differential equation. It describes that an initially thickness-free flame brush ($l_{fb} = 0$) subjected to constant input conditions of rms fluctuation velocity, integral length scale of turbulence, and laminar flame speed will first rapidly thicken due to the initially zero-magnitude decay term on the RHS of Eq. (28), followed by a declining rate of change in brush thickness over time as the magnitude of the decay term grows due to the increasing influence of self-annihilation for a thickened flame. Finally, a steady state value of brush thickness will be reached once the growth and decay terms have equal magnitudes but opposite signs. This signifies that the process has reached the dynamic equilibrium condition discussed at the introduction.

An example of the characteristic dynamic response of the flame brush thickness approaching dynamic equilibrium is given in the Fig. 13 for the fixed values of $s_L = 1$ m/s, $\bar{u}' = 1$ m/s, and $l_0 = 1$ mm.

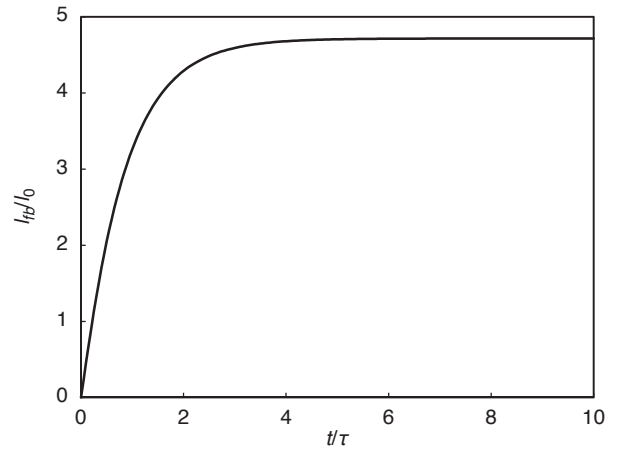


Fig. 13 Dynamic response of an initially thickness-free turbulent flame brush for constant conditions

5.4. Turbulent Flame Speed Interpretation

Although the derivations given in the preceding sections for the flame brush thickness dynamics equations have not specifically stated so, the information needed to interpret the transient motion of a rapidly propagating mean surface in the presence of turbulence from these equations has already been included. This rate of displacement rate of the flame brush thickness centerline is the same transient turbulent flame speed which was desired to be estimated with a simple physical model.

In Fig. 12 at time $t+\Delta t$, it is seen that the mean flame surface has advanced more rapidly than the leading edge of the flame when isolated from external effects. This is a consequence of the more rapid consumption of reactants near the trailing edge of a flame than near the leading edge of the flame, embodied by the decay term given in Eq. (27).

In the absence of turbulence, the mean flame surface is expected to propagate at the laminar flame speed relative to the unburned gas. In the presence of turbulence the flame brush thickens, leading to more rapid consumption and propagation near the trailing edge of the flame. A suitable approximation for the free turbulent propagation speed of the mean flame surface can therefore be given by,

$$s_T = s_L - \frac{1}{2} \frac{dl_{fb}}{dt} \Big|_{decay} \quad (29)$$

The first term on the RHS of the equation above represents the non-turbulent propagation, and the second term represents the apparent advancement of the front due to a reduction in the flame brush thickness. If the flame brush thickness is not decreasing or has a zero value, turbulent flame speed simply reduces to the laminar flame speed, as would be expected. Also note that if the flame brush thickness is strictly increasing, in the absence of other effects, the brush is still assumed to be advancing at the laminar flame speed as the increasing thickness is assumed to occur proportionally to the mean brush center line, both in the directions of leading and trailing edges equally. The leading coefficient on the second term represents that the mean flame surface is located at the flame brush centerline halfway between the leading and trailing edges of the flame, respectively. A discrete reduction in the brush thickness would therefore lead to an apparent advancement of the brush center by one half the brush thickness reduction magnitude.

Substituting in the definition for the brush thickness decay rate and rearranging gives a model for the instantaneous turbulent flame speed ratio s_T/s_L for a freely propagating flame,

$$\frac{s_T}{s_L} = \frac{1 + sec\bar{\theta}}{2} \quad (30)$$

The steady state turbulent flame speed can be found by setting the left-hand side of Eq. (27) to zero, substituting

terms into Eq. (30) and simplifying. This results in the FBTD-based equation below for the steady state turbulent flame speed,

$$\frac{s_{T,\infty}}{s_L} = 1 + \frac{6}{\pi} \frac{\bar{u}'}{s_L} \quad (31)$$

The equation above recovers the commonly proposed form of the steady state turbulent flame speed correlations embodied by Eq. (1). The historical model which most closely resembles the new FBTD model derived here is the same one proposed by Damköhler, and perhaps not unexpectedly, as many of the key interpretations and features of the stencils used in the model derivation were similarly assigned and defined.

5.5. Confined Flame Propagation

When the flame brush can freely interact with the turbulence, its dynamics are expected to be approximated by Eq. (27) and the freely propagating flame brush will ultimately reach a maximum sustainable thickness once the terms on the RHS reach equilibrium. Results for the steady state flame brush thickness for a freely propagating flame found from Eq. (28) over a range of relative turbulence intensities are shown in Fig. 14.

Other than the rapid increase near $\bar{u}'/s_L = 0$, the maximum sustainable brush thickness of a freely propagating flame is predicted to increase linearly with relative turbulence intensity according to the FBTD model. Note that even for the relatively weak turbulence conditions the flame brush is anticipated to be many times thicker than the integral length scale.

A simple conclusion that can be drawn is that a combustor needs to be of a sufficiently large geometrical scale relative to the turbulence length scale in order to contain the freely developing flame brush thickness, or the flame can no longer be truly freely propagating but rather is confined by the combustor. Representing such complicated flame-wall interaction in a simple model is unlikely to be universally valid. However, a coarse approximated criteria has been discovered from the study of the instantaneous G-equation solutions for highly confined flame development. The criteria are given below which classifies flames as those which are either freely developing or developing within confined combustor geometry.

$$\begin{aligned} \text{Freely Propagating Flame: } \frac{2l_{fb}}{d_0} &< 1 \\ \text{Confined Flame: } \frac{2l_{fb}}{d_0} &\geq 1 \end{aligned} \quad (32)$$

Where d_0 is the macro length scale of the geometry of the combustor which inhibits the free propagation of the flame. For rectangular shaped cross sections, it is found that the maximum transverse span tends to be the limitation. An approximate model for the confined flame propagation can

be made by modifying Eq. (27) as below,

$$\left. \frac{dl_{fb}}{dt} \right|_c = \zeta \frac{12}{\pi} \bar{u}' + s_L(1 - \sec \bar{\theta}) \quad (33)$$

Where ζ is the probability that the entire flame brush will be influenced by the fluctuating intensity of turbulence according to the following scheme:

Table 1 Confined flame propagation criteria

Condition:	Criteria:	ζ :
Free	$2l_{fb}/d_0 < 1$	1
Confined	$2l_{fb}/d_0 \geq 1$	$d_0/2l_{fb}$

From the values of ζ given above we can see that when a combustor's geometry is sufficiently large, Eq. (33) becomes equivalent to Eq. (27) for a freely propagating flame. While a very thick flame brush contained within an overly small combustor will experience a reduction in the magnitude of the flame brush thickness growth term due to interference with the walls of the combustor. In the table above, the interference with the combustor is considered in the transverse direction only. Normal direction interference is handled by an additional flame brush thickness destruction term which is discussed elsewhere⁽¹⁶⁾.

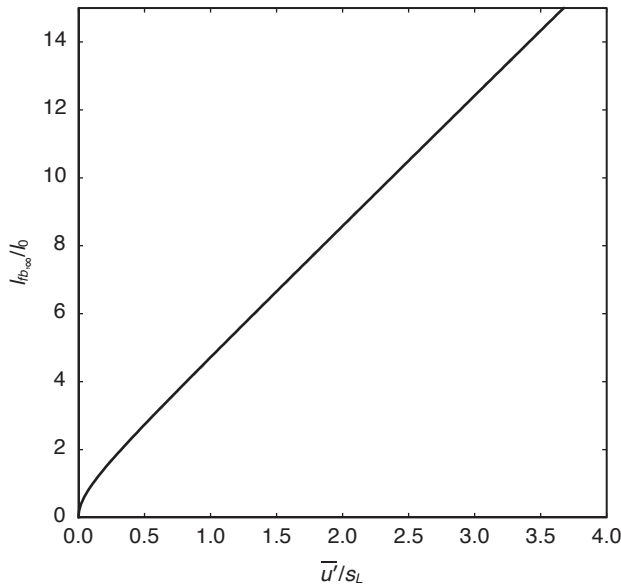


Fig. 14 Steady state flame brush thickness of a freely propagating flame versus relative fluctuation intensity

6. Comparison of FBTD vs Abdel-Gayed Database

The predicted values for a steady state, freely propagating, turbulent flame speed based on the model

derived in the previous chapter have been compared against the compiled database results of Abdel-Gayed in Fig. 15 and are shown as a solid line. We see there is a generally good agreement between results predicted from Eq. (27) and the measurements starting near $s_T/s_L = 1$ and $\bar{u}'/s_L = 0$ that increases linearly with turbulence intensity. Many of these data points were recorded from open flame burners, where the turbulence and the flame were able to interact freely without constraint by the combustor directly and is therefore expected to be well represented by Eqs. (27)-(29) for freely propagating turbulent flames. These data points have been noted in Fig. 15 by the present author as “Freely propagating flames”.

However, the compiled database results also contain a secondary nonlinear trend in the measurements which is not well represented by the equations for freely propagating flames. This so called “bending over” in the measured turbulent flame speed is commonly found in the literature. In the particular case of Fig. 15, most of these points exhibiting the nonlinear trend were collected by Petrov and Talantov⁽³⁸⁾ in the 1960s. Their experiment measured turbulent flame speeds of a stationary inverted flame cone that was stabilized by a burner within a 150 mm diameter pipe flow supplied by grid generated turbulence. They reported laminar flame speeds in the range of 0.2-0.6 m/s and Abdel-Gayed calculated the integral length scales in the range of 2.67-3.9 mm using correlations from Baines and Peterson⁽³⁹⁾.

By setting the left-hand side of Eq. (27) to zero and

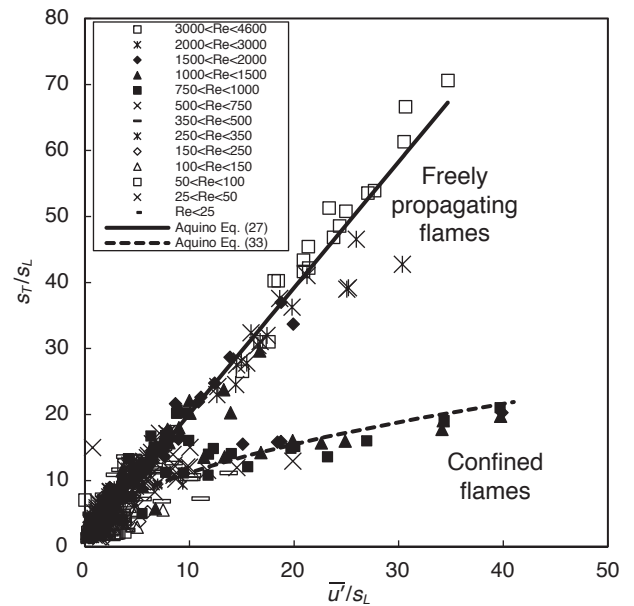


Fig. 15 Comparison of derived steady state turbulent flame speed model against turbulent flame speed measurements compiled by Abdel-Gayed

applying the given characteristic conditions of $\bar{u}/s_L = 50$, $s_L = 0.4$ m/s, and $l_0 = 3.29$ mm, we can solve for the maximum steady state flame brush thickness of a freely propagating flame expected to be found during Petrov and Talantov's experiments as approximately 632 mm. Using the criteria defined in Table 1 we find $2l_{fb}/d_0 \approx 8.5$, and therefore conclude that the flame speed measurements of Petrov and Talantov are highly confined by the combustor and are therefore not expected to be represented by Eqs. (27)-(29) for freely propagating flames. Considering that the inverted flame cone of Petrov and Talantov needs to fit within the combustor's pipe diameter of 150 mm, this analysis demonstrates that a freely propagating flame could not develop under such conditions without the geometry of the combustor interfering with the turbulent flame's propagation.

The dynamics of this type of confined flame brush thickness can be calculated from Eq. (33) using values of ζ between 1-0.118 determined by parameters found from the referenced works above. The confined turbulent flame speed ratios are then found by application of Eqs. (28) and (29). As expected from the prior discussion, the resulting confined turbulent flame speeds were found to be substantially lower than their freely propagating counterparts operating under the same conditions. These confined flame results have been added to Fig. 15 as a dashed line and the datapoints in the vicinity have been noted by the present author as "Confined flames". We can see that both the nonlinear trend versus turbulence fluctuation intensity and the magnitude of the turbulent flame speed ratio are well represented with respect to the measurements. Most of the low fidelity turbulent flame speed models have not attempted to account for these type of macro scale interference effects and therefore might incorrectly associate some aspects of the so-called "bending" phenomena to other mechanisms which are fundamentally unrelated.

7. Comparison of FBTD vs G-equation

Detailed measurements of transient flame development necessary for validating the FBTD model are not frequently present in the literature. Therefore, the dynamics of the FBTD model will be validated by comparison against calculated instantaneous G-equation solutions of resolved turbulent flame propagating for the same input physical conditions. The initialization and solving procedure used for the G-equation analysis here is the same as was discussed in Sections 4.1.-4.3.

Since the flame brush thickness is a macroscopic quantity, some method of spatially filtering the instantaneous solutions of the G-equation needs to be applied in order extract statistical information for the purposes of this comparison. In the case of simple planar flame propagation used here, a 2D sliding interrogation window technique⁽¹⁶⁾ can be implemented to extract the

relevant quantities. A depiction of this approach used to sample from the instantaneous G-equation results is shown in Fig. 16.

Processing and tabulating of the results followed the method proposed by Aquino⁽¹⁶⁾ where the three primary output quantities of interest were chosen to be the mean flame position x_0 , the flame brush thickness l_{fb} , and the flame speed ratio s_T/s_L . Several statistical quantities are reported for the G-equation results as a function of time as the detailed 3D results tend to exhibit a high degree of spatial and temporal variation. The corresponding mean values predicted by the derived FBTD model are then compared against these results in Fig. 17. Note that a more comprehensive comparison between the G-equation results and the proposed FBTD model is available by the author in the literature⁽¹⁶⁾, where the laminar flame speed, the magnitude of the fluctuation velocity, and the turbulence length scale were varied independently of one another in order to test the FBTD model's ability to represent the overall behavior of a developing turbulent flame for a much wider variety of conditions.

In the present work the comparison shown in Fig. 17 is limited to a single realization of turbulent flame propagation for the same conditions discussed at the beginning of Chapter 4. Despite the high degree of local fluctuations present in the G-equation results, the FBTD model was able to represent the mean motion of the flame surface, the dynamic response of the flame brush thickness, the resulting steady state flame brush thickness occurring at dynamic equilibrium, and finally allowed the interpretation of these behaviors into an apparent macroscopic turbulent flame speed of the mean flame surface. It was found that a similar level of agreement to the singular case presented here was also observed as other parameters were varied in general, and this suggests the simplifying assumptions used to derive the FBTD model in Chapter 5 are reasonable approximations for describing the overall dynamic behavior of a developing turbulent flame despite maintaining a relatively simple mathematical form.

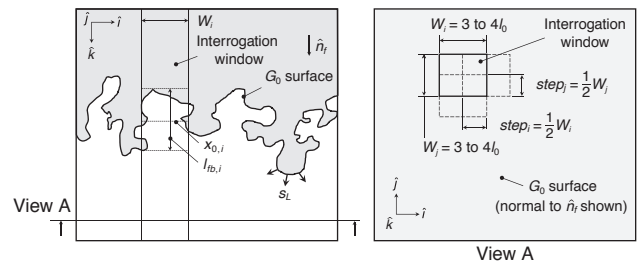


Fig. 16 Sliding interrogation window technique used for extracting statistical information from the instantaneous G-equation results

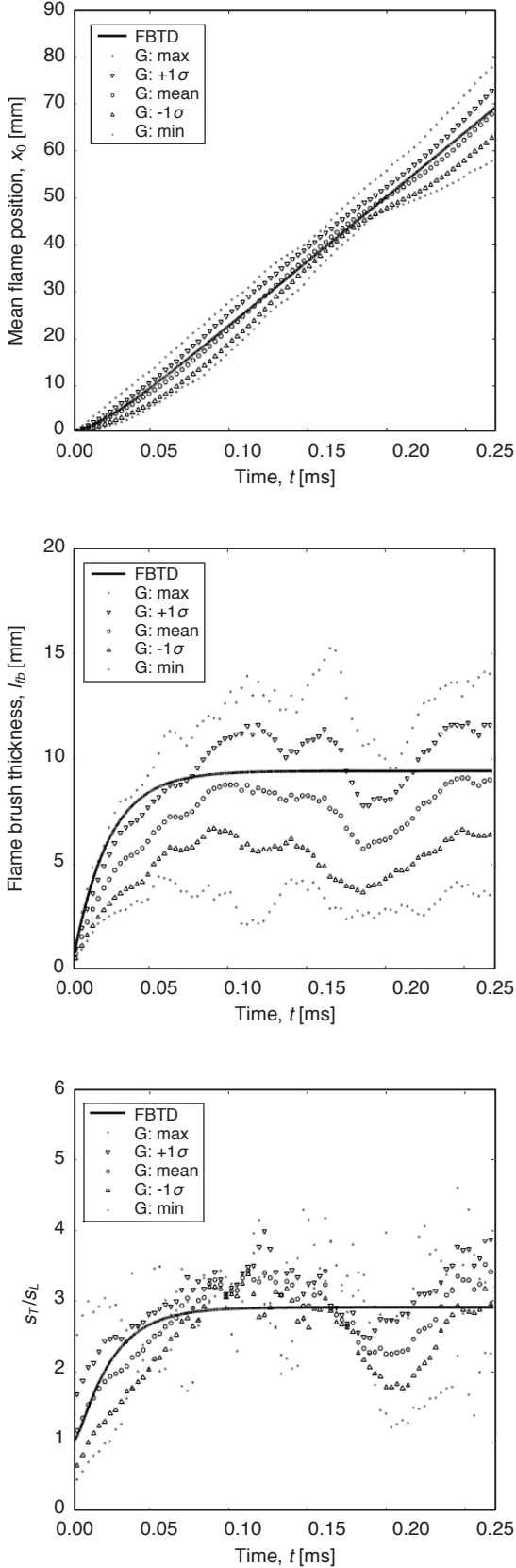


Fig. 17 Comparison of FBTD vs single realization of the G-equation for the same conditions used in Chapter 4

8. Discussion and Conclusions

The primary focus of this work was to develop a simple, physics-based model that could describe the dynamic phenomena of turbulent flame propagation. Analysis of detailed G-equation solutions lead to the concept of the dynamics of the turbulent flame brush thickness being usable as a model for the turbulent flame speed which was derived throughout Chapter 5.

Subsequent analysis found that the FBTD model gave good agreement with the dynamic behavior of the G-equation solution for many developing turbulent flames, where the details of one realization were presented in Fig. 17. Predicted steady state turbulent flame speeds were also found to compared favorably against the compiled measurement database of Abdel-Gayed across a wide range of turbulence intensity and experimental arrangements.

An interesting consequence of FBTD theory is the observation that large spatial dimensions relative to the turbulence length scales are necessary for a turbulent flame to develop to the peak turbulent flame speeds possible under given conditions. Analysis of the historical record indicates that the experimental apparatus used in combustion measurements are occasionally of insufficient dimension to prevent interference with the developing turbulent flame brush, and so although a sampled turbulence intensity may be quite high, the measured turbulent flame speed can be found to be significantly lower than those for otherwise similar operating conditions when either a larger combustor or smaller turbulence length scales are present.

Turbulent flame speed models do not typically contain criterion for the kind of spatial dimension requirement discussed above, and so many models present in the literature may incorrectly associate some aspects of the relationship between the turbulent flame speed and the turbulence intensity to other mechanisms which are fundamentally unrelated. This observation was applied in the analysis of Petrov and Talantov's measurements given in Chapter 6 and the significance of the described effect can be seen in Fig. 15.

8.1. The FBTD Model

$$\left. \frac{dl_{fb}}{dt} \right|_c = \zeta \frac{12}{\pi} \bar{u}' + s_L(1 - \sec \bar{\theta}) \quad (34)$$

$$\bar{\theta} = \tan^{-1}(l_{fb}/l_0) \quad (35)$$

$$\frac{s_T}{s_L} = \frac{1 + \sec \bar{\theta}}{2} \quad (36)$$

The equations listed above are collectively referred to as the FBTD model. Equation (34), along with the definition provided by Eq. (35), are intended to be used for predicting the transient turbulent flame brush thickness for a given

laminar flame speed, rms fluctuation velocity, and integral length scale of turbulence. Equation (36) then interprets the dynamics of the turbulent flame brush thickness to allow calculation of the apparent instantaneous turbulent flame speed.

It is anticipated that the FBTD model may find utility in the analysis of the dynamics of a developing turbulent flame present, like those present in many practical combustion devices such as the common internal combustion engine.

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