

Robust Optimization for Windmill Airfoil Design under Variable Wind Conditions

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ABSTRACT

Robust design of windmill airfoils is carried out in the presence of uncertain environmental conditions. The formulation follows a multi-objective optimization approach, where the mean aerodynamic performance is maximized while sensitivity to changes in wind conditions is kept to a minimum. Due to the complex nature of the physical processes involved, a non-intrusive collocation approach is used to quantify uncertainties, coupled with a RANS solver for the fluid flow. A robust design optimization framework is thus assembled, where Genetic Algorithms are used to iteratively evolve section parameters toward optimal designs. Expectations and variances are computed with a Smolyak sparse grid approach combined with nested Clenshaw-Curtis quadrature, leading to a minimization of the number of deterministic solutions needed. The results show the potential applicability of the proposed methodology to the robust design of industrial devices.

1. Introduction

Wind power generation, the production of electric power by conversion of wind energy into propeller rotation, is increasingly seen as an attractive energy source.

The ability to achieve high conversion efficiency under changing wind conditions is crucial to this technology.

Optimization should guarantee a stable performance while maximizing the rotation thrust or, in other words, should be formulated as a robust process.

However, due to the significant computational cost associated with traditional Monte Carlo-like strategies used in conjunction with CFD simulations, design optimization is often carried out for specific environmental conditions.

The objective of this study is twofold:

- To investigate and show the increase in computational cost involved in the transition from traditional to robust optimization for windmill airfoil section profiles; and
- To explore efficient optimization methods.

2. Problem Formulation

The velocity triangle of a windmill airfoil section illustrated in Fig. 1 shows how the change in wind velocity

V_a results in changes in the angle of attack and inflow velocity relative to the airfoil section.

Two optimization problems are analyzed in the present study. The first approach is using a traditional optimization strategy where no variation of inflow wind conditions is considered. It is formulated as a single objective optimization, maximizing a measure of aerodynamic efficiency, i.e., the lift-to-drag coefficient ratio C_L/C_D .

Robust optimization is successively investigated, where disturbances in the inflow angle and Mach number are associated with uniformly distributed random variables

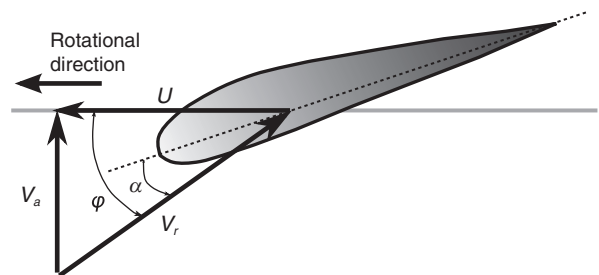


Fig. 1 Schematic representation of velocity triangle for windmill airfoil section

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α and M , respectively. The average performance is thus maximized over a range of wind conditions, while minimizing the associated variance, i.e., the sensitivity to stochastic environmental changes.

Incompressible flow conditions are assumed throughout; the flow field is computed using a RANS solver at constant Reynolds number, where the airfoil geometry and wind conditions are determined by realizations of the parameters. Assume the airfoil cross-section geometry is determined by a vector $\xi \in D$ of parameters, where $\xi = \{\xi_1, \dots, \xi_{np}\}$, $\xi_i^l \leq \xi_i \leq \xi_i^u$ and D is the compact space of feasible airfoil configurations; ξ_i^l, ξ_i^u are the lower and upper bounds, respectively, for the generic parameter ξ_i . Moreover, consider a probability space (Ω, F, P) in which Ω is the sampling space, F is the σ -algebra of subsets of Ω , and P defines a probability measure on F . A vector of two independent and identically distributed random variables with joint probability density function $\rho(y) = \rho_\alpha(\alpha) \rho_M(M) : \mathbb{R}^2 \rightarrow \mathbb{R}^{\geq 0}$ is indicated by $y = (\alpha, M)$ with $\alpha, M : \Omega \rightarrow \mathbb{R}$. Realizations of the stochastic vector y are denoted by $y^{(i)} = (\alpha^{(i)}, M^{(i)})$, $i = 1, \dots, n_i$, where n_i is the total number of samples. Consider $f : D \times \Omega \rightarrow \mathbb{R}$ a function mapping design and stochastic parameters into lift-to-drag coefficient ratios. We also stress that every evaluation of f requires prior sampling of design parameters ξ and environmental variables $y^{(i)} = (\alpha^{(i)}, M^{(i)})$ followed by a RANS simulation. Design optimization can be formulated using two different approaches:

- (a) Without including the uncertainty in wind conditions (traditional optimization).

Find $\xi^* \in D$ such that

$$\xi^* = \arg \max_{\xi \in D} f(\xi, y^{(i)}) \quad (1)$$

- (b) Including the environmental uncertainty (robust approach).

Find $\xi^* \in D$ such that

$$\mathcal{M}(\mathbb{E}\{f(\xi^*, y)\}, \sigma\{f(\xi^*, y)\}) \geq \mathcal{M}(\mathbb{E}\{f(\xi, y)\}, \sigma\{f(\xi, y)\}) \quad \forall \xi \in D \quad (2)$$

where $\mathbb{E}\{\cdot\}$ and $\sigma\{\cdot\}$ denote the expectation and standard deviation operator, expressed as:

$$\mathbb{E}\{f(\xi, y)\} = \int_{M_i} \int_{\alpha_i}^{\alpha_u} f(\xi, y) \rho_\alpha(\alpha) \rho_M(M) d\alpha dM \quad (3)$$

$$\sigma\{f(\xi, y)\} = \left(\int_{M_i} \int_{\alpha_i}^{\alpha_u} (f^2(\xi, y) - (\mathbb{E}\{f(\xi, y)\})^2) \rho_\alpha(\alpha) \rho_M(M) d\alpha dM \right)^{1/2} \quad (4)$$

and $\mathcal{M} : \mathbb{R}^2 \rightarrow \mathbb{R}$ is a multi-objective decision criterion. In other words, the solution of the robust optimization problem maximizes the mean lift-drag ratio across variable environmental conditions while minimizing the associated standard deviation, i.e., the sensitivity of the airfoil efficiency to alterations in the angle of attack and Mach number. Here $[\alpha_l, \alpha_u]$ and $[M_l, M_u]$ are elementary event intervals associated with the inflow angle α [deg] and Mach number M , respectively. In practice, we adopt

$$[\alpha_l, \alpha_u] = [\alpha_0 - 2, \alpha_0 + 2], \alpha_0 = 10 \quad (5)$$

$$[M_l, M_u] = [M_0 - 0.05, M_0 + 0.05], M_0 = 0.25 \quad (6)$$

As formulated in Equation (2), the problem is essentially a multi-objective optimization whose solution is generated with a trade-off between the two objective functions, i.e., the average performance and sensitivity to environmental conditions. Note that different choices of $\mathcal{M}$ lead to different optimal solutions. For example, given two objectives $o_1, o_2 \in \mathbb{R}$, $\mathcal{M}(o_1, o_2) = o_1$ generates the solution with the maximum average efficiency, $\mathcal{M}(o_1, o_2) = -o_2$ that with minimum sensitivity, while $\mathcal{M}(o_1, o_2) = \sqrt{(o_1/n_1)^2 + (o_2/n_2)^2}$ might be chosen as a compromise. Here, the normalization factors n_1 and n_2 account for cases where objectives are not homogeneous or provided in different units.

3. Airfoil Representation

The PARSEC⁽¹⁾ representation is used to define a parametric airfoil profile.

In this representation, the upper and lower section curves are expressed by polynomials of the following form:

$$z = \sum_{i=1}^6 a_i x^{(i-1/2)} \quad (7)$$

where the section is defined in the (x, z) plane.

The coefficients a_i are determined from the geometric parameters $\xi \in D$ (design variables), illustrated in Fig. 2. Eleven design variables are selected: leading edge radius, upper and lower crest locations and curvatures, trailing edge coordinates (at $x = 1$), thickness, direction angle and wedge angle. Design variables and corresponding ranges are represented in Table 1. As PARSEC tries to minimize the number of parameters needed to generate wing profiles of practical interest in applications, it is useful to consider the possible interaction between some of these parameters. An example is illustrated in Fig. 3, where different values are applied to the parameters z_{TE} and $z_{lo} + z_{up}$, respectively.

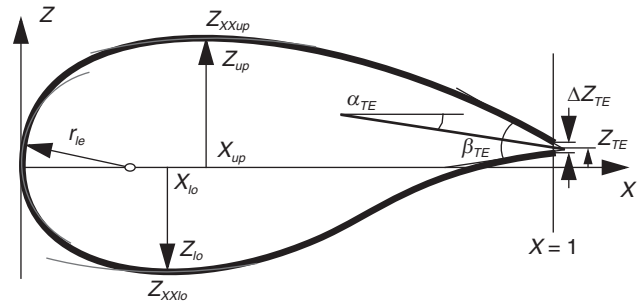
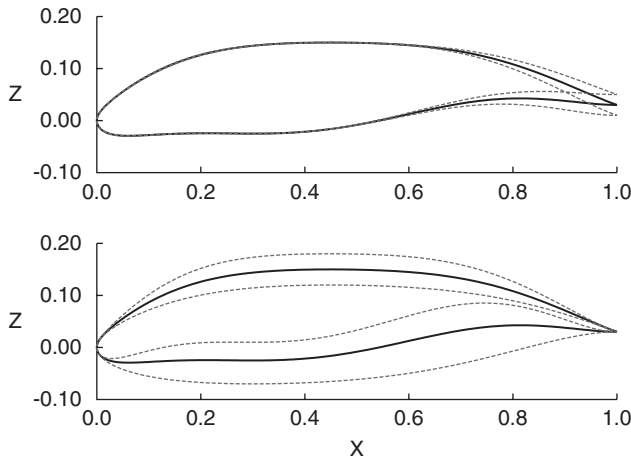


Fig. 2 Graphical representation of input parameters for PARSEC

Table 1 List of parameters used in PARSEC and α , M

ID	Description	ξ_l	ξ_u
r_{le}	Radius of leading edge	0.005	0.02
x_{up}	X at crest of upper surf.	0.3	0.7
z_{up}	Z at crest of upper surf.	0.12	0.18
$z_{xx,up}$	Curv. of upper surf.	-0.4	0.0
x_{lo}	X at crest of lower surf.	0.2	0.6
z_{lo}	Z at crest of lower surf.	-0.07	0.02
$z_{xx,lo}$	Curv. of lower surf.	0.3	0.9
z_{TE}	Z at trailing edge	-0.01	0.05
α_{TE}	Camber gradient at trailing edge	-13.0	-3.0
Δz_{TE}	Thickness at trailing edge	0.0	0.0
β_{TE}	Wedge angle at trailing edge	4.0	8.0
α	Angle of attack	8.0	12.0
M	Mach number	0.2	0.3

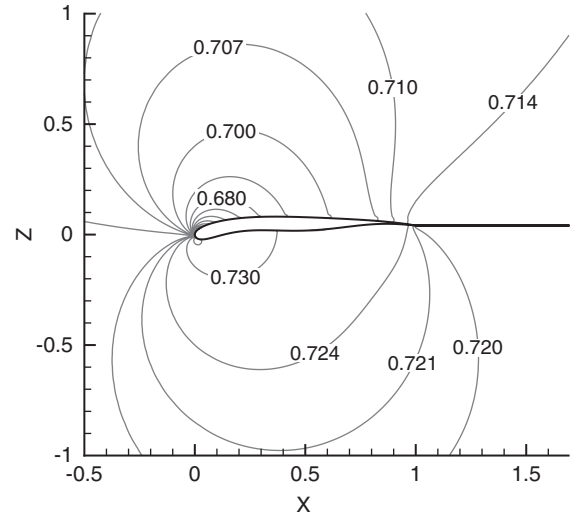
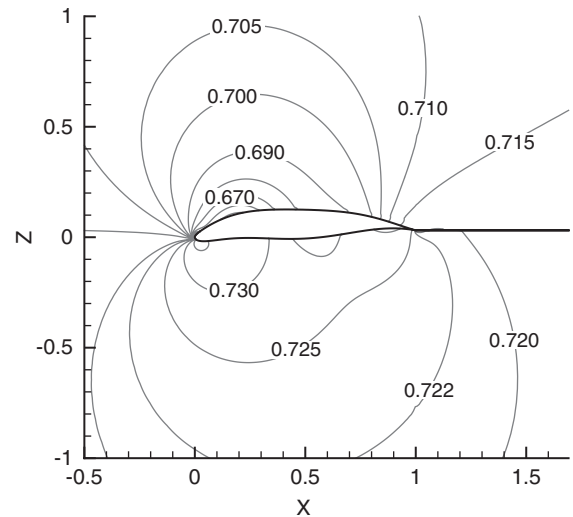
Fig. 3 Change in airfoil configuration as a result of adjusting z_{TE} (top) or $z_{lo} + z_{up}$ (bottom)

4. Computation of Lift and Drag, Preliminary Optimizations

Once the airfoil profile is determined by the parameter set ξ , the solution to the two-dimensional incompressible Navier-Stokes equation is sought using a Reynolds number equal to 10^5 . An automatic meshing procedure is first carried out starting from the profile generated by PARSEC. The CFD solver⁽²⁾ uses a delta form implicit finite difference method⁽³⁾. A 3rd-order Chakravarthy-Osher TVD limiter⁽⁴⁾ is also used for advection together with the Menter k- ω SST fully transitional turbulence model⁽⁵⁾.

As a preliminary step, the windmill profile was optimized under known wind conditions $(\alpha, M) = (10, 0.25)$. The resultant shape is shown in Fig. 4. A very thin airfoil is generated, as expected, exhibiting a pronounced curvature of the leading edge in the direction of the selected angle of attack. This solution, although characterized by very high levels of aerodynamic efficiency, shows an insufficient

section modulus, and thus lacks structural strength. An alteration to the initial design parameter space D is thus required to fulfill both aerodynamic and structural feasibility. The modified “constrained” optimal solution is represented in Fig. 5. In practice, the initial range for z_{up} equal to $[0.08, 0.18]$ has been modified to $[0.12, 0.18]$, resulting in the section thickness range to change from $[0.06, 0.25]$ to $[0.10, 0.25]$, respectively.

Fig. 4 Pressure levels for unconstrained optimal airfoil design under certain wind conditions $(\alpha, M) = (10, 0.25)$ Fig. 5 Pressure levels for constrained optimal airfoil design under certain wind conditions $(\alpha, M) = (10, 0.25)$

5. Robust Optimization Methods

Shape optimization of windmill airfoils is performed by means of Genetic Algorithms (GA).

This family of methodologies seeks optimal solutions by

selectively creating successive generations of individuals. Each individual represents a design configuration with chromosomes ξ , i.e., the 11 section design parameters. Each individual is also associated with a set of environmental conditions, that is, realizations of angle of attack and Mach number $y^{(i)} = (\alpha^{(i)}, M^{(i)})$, $i = 0, \dots, n_i$. These locations are carefully selected to facilitate the computation of statistics via multivariate quadrature. This differentiates a robust approach from traditional optimization, where a single evaluation of f is sufficient for every individual in the population.

Now we assume that $f(\xi^*, y)$ takes the form

$$f(\xi^*, y) \approx \sum_{i=0}^P \alpha_i(\xi^*) \phi_i(y) \quad (8)$$

as a finite linear combination of tensor product orthogonal polynomials of the random vector f (polynomial chaos expansion). The quantities $\mathbb{E}\{f(\xi^*, y)\}$ and $\sigma\{f(\xi^*, y)\}$ can be computed by numerical integration as follows:

$$\mathbb{E}\{f(\xi^*, y)\} \approx \sum_{j=1}^{n_i} \omega_j f(\xi^*, y^{(j)}) \quad (9)$$

$$\sigma\{f(\xi^*, y)\} \approx \left\{ \sum_{j=1}^{n_i} \omega_j [f(\xi^*, y^{(j)}) - \mathbb{E}\{f(\xi^*, y)\}]^2 \right\}^{1/2} \quad (10)$$

where $\sum_{j=1}^{n_i} \omega_j = 1$. Here the n_i quadrature locations are the zeros of the selected tensor product polynomial family $\phi_i(y)^{(6)}$. Optimal convergence to the statistics of sufficiently smooth stochastic responses is obtained by employing polynomial families orthogonal to the input probability measures. Our choice of adopting Clenshaw-Curtis quadrature^{(7), (8)} locations translates into expanding f using Chebyshev polynomials. For applications where f is not known in advance and arbitrary input probability measures can be specified, the good convergence properties of Chebyshev approximants give us a good compromise to be implemented in a general framework. For every computed generation, the steps performed by GA are highlighted in Fig. 6.

The computational cost of evaluating multivariate integrals, like $\mathbb{E}\{f(\xi, y)\}$, with given accuracy leads to a dramatic increase of polynomial terms (or numerical integration points) for a corresponding increase in the number of stochastic input variables. This fact is generally referred to as the “curse of dimensionality”; a typical trend is shown in Table 2 (Top). We also stress that every evaluation of $f(\xi^*, y^{(i)})$ requires a complete RANS simulation. Thus, the number of deterministic realizations must be kept to a minimum if robust optimization is to be performed in a reasonable period of time. In this study, we adopt a Smolyak sparse grid⁽⁹⁾ approach together with nested Clenshaw-Curtis quadrature as a possible mitigation of this phenomenon (Table 2 Bottom). A two-dimensional Smolyak sparse grid with 13 points is used in the present study. Approximation order 0, 1 and 2 are thus recovered with 1, 5 and 13 quadrature points, respectively, allowing rough estimates of convergence and accuracy to be computed. Figure 7 shows the selected quadrature grid in the $\alpha - M$ plane. Finally, we note that this study employs a non-intrusive approach that uses an

Table 2 Increase in multivariate quadrature points with dimensionality for fixed one-dimensional polynomial accuracy

Dimension	1	2	3	4	5	n
Full grid	5	25	125	625	3125	5^n
Smolyak sparse grid	5	13	33	89	253	$3^n + 2n$

unmodified deterministic CFD solver even if stochasticity has been injected into the equations. This approach is justified by the fact that the accuracy of the RANS solution is largely constant for the selected parameter ranges.

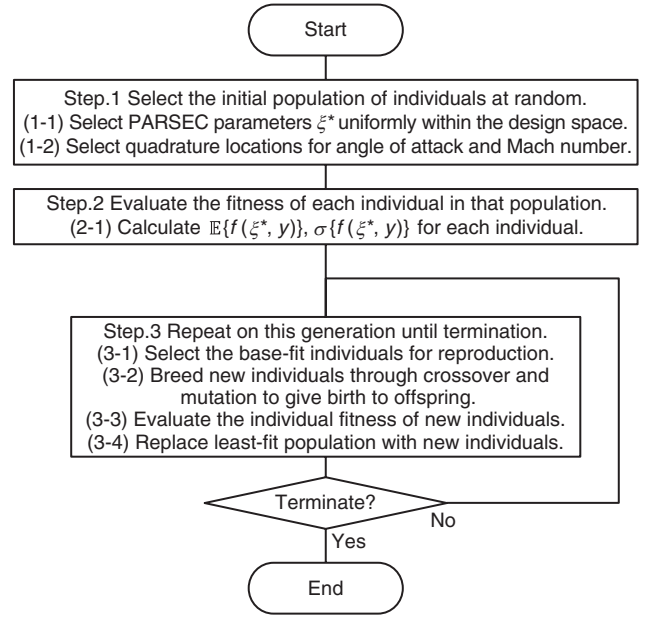


Fig. 6 Genetic Algorithm

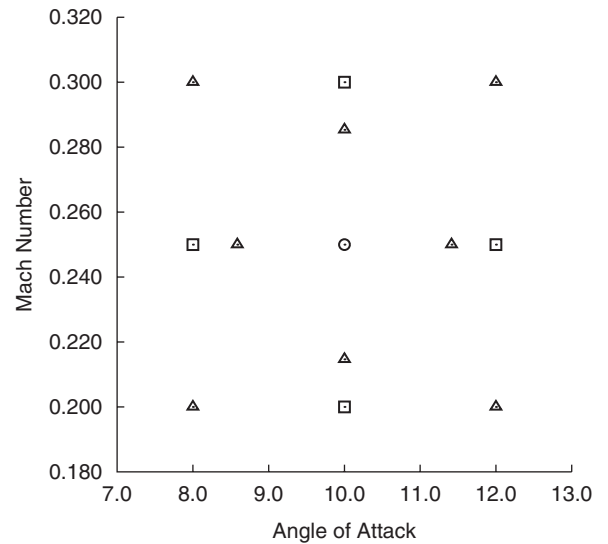


Fig. 7 Two-dimensional Smolyak sparse tensor quadrature grid up to order 2 accuracy. Level 0 ($\circ$), Level 1 ($\square$) and Level 2 ($\triangle$) incremental grids are shown.

Table 3 Sensitivity of optimal design to GA parameters

Population size	20	50	100
Optimal C_L/C_D	38.20	37.70	39.22
Cross probability	0.2	0.4	0.6
Optimal C_L/C_D	37.94	39.23	39.22
Flip probability	0.002	0.004	0.02
Optimal C_L/C_D	39.22	39.58	38.22

6. Optimal Windmill Airfoils

First, a sensitivity analysis is performed to assess how the optimal design is influenced by some of the GA parameters, i.e., population size, cross probability, and flip probability.

This is performed for fixed environmental conditions $(\alpha, M) = (10, 0.25)$; results are illustrated in Table 3. For a sufficient population size, sensitivity to the GA parameters has a limited effect on the optimal solution.

Two optimization tasks denominated “traditional” and “robust” are then carried out, generating optimal designs ξ_T^* and ξ_R^* , respectively.

The location of the traditional optimal design in the $(C_D, 1/C_L)$ plane is highlighted in Fig. 8. After the set of parameters ξ_T^* is found giving the maximum C_L/C_D ratio for fixed environmental conditions $(\alpha, M) = (10, 0.25)$, an uncertainty propagation analysis is carried out leading to $\mathbb{E}\{f(\xi_T^*, y)\} = 30.46$ and $\sigma\{f(\xi_T^*, y)\} = 2.53$ (Table 4, upper).

Some of the generations produced by robust optimization are also shown in Fig. 9. Optimal designs have been reported that correspond to various metrics M , i.e., maximum expected efficiency, minimum standard deviation

Table 4 Comparison of results for traditional and robust optimizations

	Design	Statistic	C_L/C_D
Traditional optimizations	Max C_L/C_D	Mean	30.46
		SD	2.53
		SK	-0.45
Robust optimizations	Maximum average efficiency	Mean	32.68
		SD	1.64
		SK	-0.40
	Minimum combined metric	Mean	15.96
		SD	0.24
		SK	-0.04
	Minimum sensitivity	Mean	2.35
		SD	0.10
		SK	0.12

SD: Standard Deviation, SK: Skewness

and best compromise. Values of $\mathbb{E}\{f(\xi_R^*, y)\}$ and $\sigma\{f(\xi_R^*, y)\}$ for designs selected according to the above metrics are also reported in Table 4 (bottom).

Note that first-, second-, and also third-order statistics have been included in the analysis.

Moreover, a negative skewness is observed for most reported designs, showing an asymmetry of the pdf of f with a longer tail towards values of smaller efficiency. This provides an even stronger motivation for including variance minimization as a further optimization objective.

Robust optimal designs with minimum standard deviation and best compromise metrics are much less sensitive to the environmental conditions than traditional optima.

However, average performance is also significantly

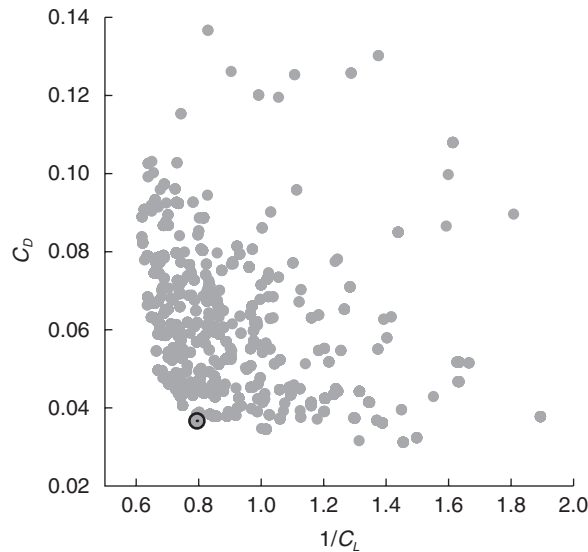


Fig. 8 Individuals generated by GA for single-objective optimization not accounting for variability in wind conditions. The optimal constrained design (○) is highlighted.

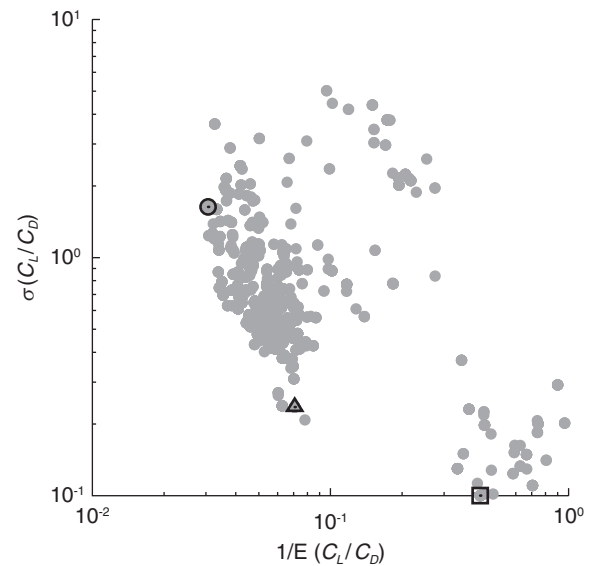


Fig. 9 Efficiency-sensitivity tradeoff resulting from robust optimization. Designs with maximum average efficiency (○), minimum combined metric (△) and minimum sensitivity (□) are highlighted.