

Development of Real-time Aerodynamic Design System

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ABSTRACT

A real-time aerodynamic design system has been developed. The new system makes possible real-time computation of the aerodynamic performance and shape sensitivity of passenger cars. The system is made up of technological elements including approximation modeling, categorizing, extraction of feature values, sensitivity analysis and automatic database update. It realizes real-time computation of aerodynamic performance through the combination of approximation modeling and categorizing technologies, computation of shape sensitivity by means of on-line sensitivity calculations, and continuous increase in accuracy by means of automatic learning.

Computational results for Computational Fluid Dynamics (CFD) and the real-time aerodynamic design system were compared in order to evaluate the performance of the system, verifying its effectiveness and ability to reduce computation time.

1. Introduction

The demand for enhanced automotive fuel efficiency has conspicuously increased the importance of reducing air resistance. Achieving a balance between aerodynamic performance and an attractive exterior form, a factor that has a considerable effect on commercial appeal, is therefore an issue.

Recently, wind tunnel experiments and CFD are used to evaluate aerodynamic performance. Wind tunnel experiments necessitate extensive investments of time and funds, for the construction of the facilities, the production of body models, and so on. By contrast with wind tunnel experiments, CFD does not require large-scale facilities. In addition, dramatic increases in computer speed have reduced computation time, bringing CFD to the stage of practical use as a design tool. Significant reductions in the person-hours spent on preparation for computation, realized through advancement of the algorithms for the generation of computational grids, has greatly increased expectations on CFD as a method of evaluation of aerodynamic performance in the initial stages of automotive body design. These initial stages of the body design process are characterized by a significant amount of change in the

shape of the body and a large number of trials, and there is a strong need to reduce computation time. Because CFD linearizes and solves nonlinear conservation of mass, momentum and energy equations, the governing equations of fluid phenomena, using computational grids formed in space, an increase in the number of computational grids also increases computation time. At present, even when the latest parallel processors are employed, CFD computation of a grid resolution adequate for the reproduction of the aerodynamic performance of a vehicle body takes several hours. In addition, the results of CFD computations are physical quantities such as flow velocity and pressure, and the challenge represented by the process of deciding on guidelines for understanding the phenomena and modifying performance based on these results can be pointed to as an issue other than computation time presented by the method.

In an attempt to resolve these issues, a real-time aerodynamic design system has been developed with statistical approximation models based on machine learning (“approximation models” below) and online sensitivity analysis as the main element technologies. This paper will discuss the technologies making up the system and system performance, and will provide details of its usefulness together with relevant data.

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screen until the desired aerodynamic performance values are obtained.

2.2. Data Management

The data accumulated in the real-time aerodynamic design system database is formulated by the process of generation of the database of previous computations, described in Section 2.1.1, and the process of automatic performance-boosting accuracy increase, described in Section 2.1.2. The open source data management software PostgreSQL is used for the management of this data. The real-time aerodynamic design system database is made up of polygon shape data, feature value data, and performance value data. A Database Management System (DBMS), which will be discussed from the next section onwards, was developed and put into operation to handle processing, including the interfaces used for writing to the database and other procedures. The use of the DBMS has helped to enable efficient data management and searching within the database. The DBMS is also able to respond to system elements including approximation models, categorizing, the automatic accuracy increase function, and sensitivity analyses.

2.2.1. Structure of Database

Like a standard database, the real-time aerodynamic design system database is not limited to aerodynamic performance values. Figure 4 shows the three-layered database structure. The system database is made up of design projects and design processes. Here, design projects play the role of grouping different design processes.

Design projects are generated based on the stage of the development process and the name of the model under development. Data including actual feature values and aerodynamic performance is stored in design processes together with the stage of development, the body data, and the user's user name. The various data stored in design processes are linked to body surface shape data defined by triangular unstructured grids.

2.2.2. Category management

The data in the database is classified into several categories based on similarity of shape. These categories

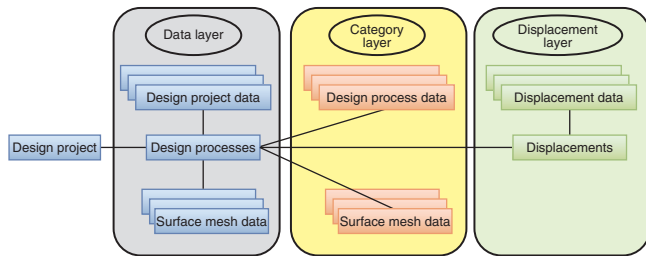


Fig. 4 Visualization of the three-layered database structure for managing design categories within the database

are generated entirely automatically in order to increase the accuracy and reliability of the approximation models. The database is provided with hypothetical category boundaries for classification into categories.

Each time data is added, the category layers are reconstructed. Computation of the degree of displacement in design data and the learning of approximation models are conducted for each category. These processes are also rerun each time the category layers are reconstructed.

3. Approximation Models

The approximation models used in the real-time aerodynamic design system are functions that output aerodynamic performance with feature values extracted from body shapes as input values. Multiple parameters are incorporated in the approximation models. These values are estimated by machine learning using data in the database. The system uses Kriging^{(3), (4)} and variational Bayesian sparse regression (VBSR)⁽⁵⁾ as approximation modeling methods. Overviews of these modeling methods and their characteristics are presented below.

3.1. Kriging

Because the Kriging method makes it possible to create high-accuracy approximation models, it is widely applied in engineering fields. Equation (1) shows the Kriging approximation equation.

$$\hat{y} = C_0 + \sum_{n=1}^N C_n f_n(\mathbf{x}_p) . \quad (1)$$

Here, $\hat{y}$ is the approximated performance value, $\mathbf{x}_p$ is the feature value for the prediction data, N is the number of sample points, and C_n and $f_n(\mathbf{x}_p)$ are an arbitrary coefficient and the radial basis function, respectively. Equation (1) shows that predictions via the Kriging method are the weighted sum of the radial basis functions (the second term on the right-hand side of the equation) added to bias C_0 . Figure 5 shows a conceptual diagram of Kriging method prediction.

Equations (2) and (3) show the radial basis functions used in the Kriging method.

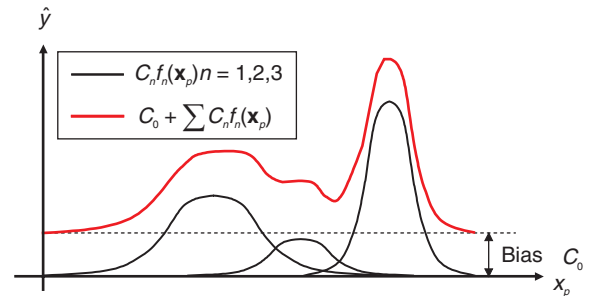


Fig. 5 Conceptual figure of Kriging

$$f_n = \mathbf{R}(\mathbf{x}_p, \mathbf{x}_m), \quad (2)$$

$$\mathbf{R}(\mathbf{x}_i, \mathbf{x}_j) = \prod_{d=1}^D e^{-\theta^d |x_i^d - x_j^d|^{\rho^d}}. \quad (3)$$

Here, $\mathbf{x}_m$ is the feature value for learned data, d is the feature value index, D is the number of feature values, and $\mathbf{R}(\mathbf{x}, \mathbf{x})$ is the spatial correlation function (SCF). ρ and θ in $\mathbf{R}(\mathbf{x}, \mathbf{x})$ are, respectively, parameters that decide the extension and smoothness of the radial basis function. Figure 6 shows changes in the radial basis function with changes in ρ and changes in θ . The left-hand graph shows that the higher the value of ρ , the more the radial basis function tends towards a smooth convex shape. The right-hand graph shows that the higher the value of θ , the narrower the range in which the effect of the corresponding learning points appears.

ρ and θ are unknown parameters for which only the number of feature values exists, and it is necessary to estimate them. The parameters are estimated based on maximum likelihood estimation (MLE). To help prevent convergence on a local solution, a global search using a genetic algorithm (GA) is first conducted, following which values are converged using hill-climbing, a local search method. The Kriging method has a greater number of parameters than other approximation modeling methods, and it offers a high degree of freedom in approximation functions. For this reason, it can be expected to construct high-accuracy models. However, the extended learning time necessary for the estimation of parameters represents an issue. Parameter estimation time is strongly dependent on the amount of learning data.

3.2. VBSR

VBSR is a method used in the discrimination of brain activity, and possesses the ability to automatically determine valid feature values for prediction during model learning. Because of this, it is also able to deal with the issue of an extremely large number of feature values (several thousand or more). Equation (4) is the approximation equation for VBSR.

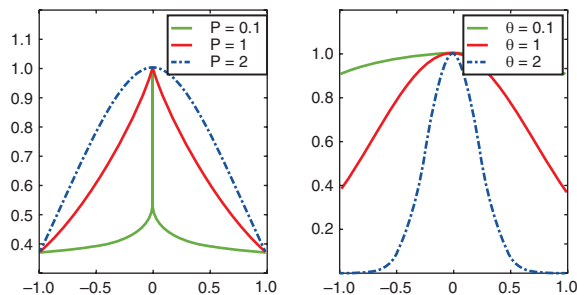


Fig. 6 Correlation of radial basis function with varying ρ (left), θ (right)

$$\hat{y} = \mu + \sigma y \sum_{i=1}^{D_{act}} w_i x_i. \quad (4)$$

Here, μ is the average value of learning data, σy is the standard deviation of learned data, i is valid feature values, D_{act} is the number of valid feature values, and w is the weighting of each feature value. Equation (4) shows that in VBSR, predictions are the sum of bias (μ ; the first term on the right-hand side) and figures obtained using the linear weighted sum method. The approximation equation is a simple linear combination of feature values and weighting. Figure 7 shows a comparison of the Kriging and VBSR methods for a one-dimensional problem. As the graphs show, the Kriging method generates a curved line that passes through almost all of the learned data, while the VBSR method generates a straight line that maintains an undeviating distance from all of the learned data.

VBSR uses Bayesian estimation for parameter estimation. Like the maximum likelihood estimation that is used for parameter estimation in the Kriging method, Bayesian estimation is a statistical parameter estimation method, and because it treats parameters as probabilistic variables, it is able to respond to factors such as bias in the data. In addition, a major feature of Bayesian estimation is its ability to estimate parameters of parameters (hyper-parameters). Utilizing this feature of Bayesian estimation, the estimation of hyper-parameter α , which judges whether w is valid for prediction, in addition to parameter w , in the VBSR method has reduced the number of parameters involved in model learning. α is known as an automatic relevance determination prior (ARD).

Model learning in VBSR avoids treating w and α as unknown quantities simultaneously, using the variational Bayes method to alternately maximize the posteriori distribution of each, and continuing to update the values of the respective parameters until the values converge. When w is updated, if there is a value of w below the threshold value, it is excluded from model learning. This is a mechanism for reducing the number of feature values in VBSR. Learning time for the VBSR method is extremely short compared to the Kriging method. In addition, like the Kriging method, parameters are determined automatically, with no necessity for experience on the part of the user. Learning time in VBSR is strongly dependent on the number of feature values.

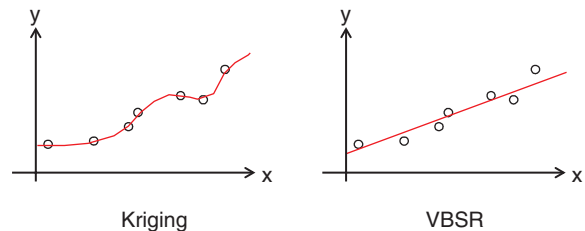


Fig. 7 Comparing phenomena of Kriging and VBSR

The real-time aerodynamic design system uses either the Kriging or the VBSR method depending on the prediction domain. In order to reduce learning time for the Kriging method, the system uses feature values reduced in number by the VBSR method for Kriging learning.

4. Categorizing

Figure 8 shows an overview of categorizing. Categorizing is broadly divided into Clustering, shown on the left, and Classifier, shown on the right. Clustering involves classification of data by the grouping of large amounts of data prepared in advance into groups of similar data called clusters. The prediction of categories for new data in relation to already classified data is called Classifier. Clustering methods include self-organizing map (SOM)⁽⁶⁾ and K-means⁽⁷⁾ and Classifier methods include neural network (NN)⁽⁸⁾, sparse logistic regression (SLR)⁽⁵⁾ and support vector machine (SVM)⁽⁹⁾.

4.1. Categorizing in the System

In order to increase the accuracy of aerodynamic performance prediction by means of the use of approximation models, the real-time aerodynamic design system organizes shapes with a high degree of similarity into clusters, and generates approximation models for each cluster. Figure 9 shows images of the application of the categorizing technology in the system. The image on the left shows an example of the generation of an approximation model for all the data in the database. The image on the right shows an example of the generation of approximation models for each cluster following the organization of the data in the database into clusters based on shape.

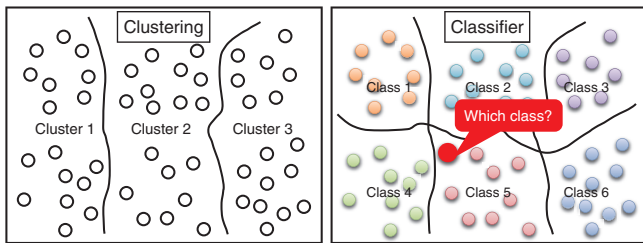


Fig. 8 Clustering and Classifier

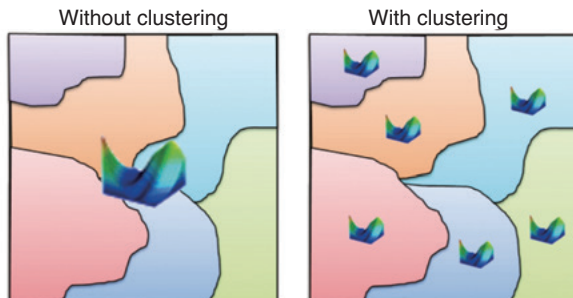


Fig. 9 Application of categorizing technique

Figure 10 shows an image of the data accumulated in the real-time aerodynamic design system database. In actual operation, classes are generated for each baseline body shape. The data in each class is generated from the baseline shape data via the method discussed in Section 2.1.1.

When new shape data is input, the appropriate class is determined by means of SLR. SLR is a method that expands logistic regression with Bayes estimation, and is applied as a method of assigning classes, with the weighted estimates for the discrimination of classes output by the method as probabilities of class membership.

4.2. Treatment of Category Boundaries

Figure 11 shows an image of class boundaries. When all data is classified into multiple classes, and predictions are made using the approximation models generated for each class, predictive accuracy at the class boundaries becomes an issue.

Figure 12 shows an image of the treatment of class boundaries in the real-time aerodynamic design system.

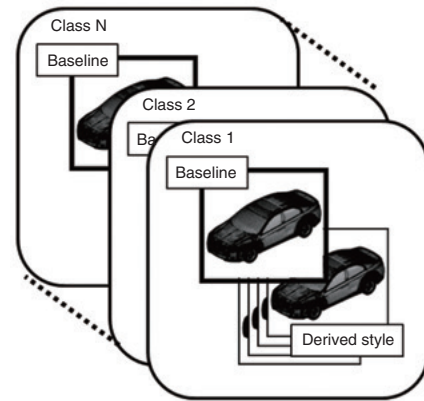


Fig. 10 Data structure image of DB

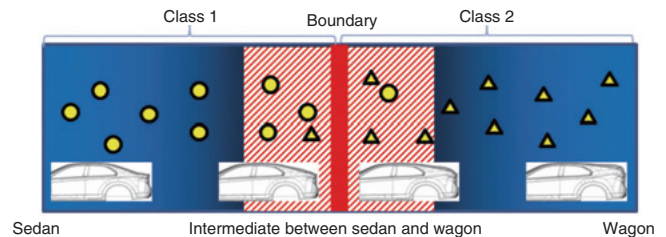


Fig. 11 Image of category boundary

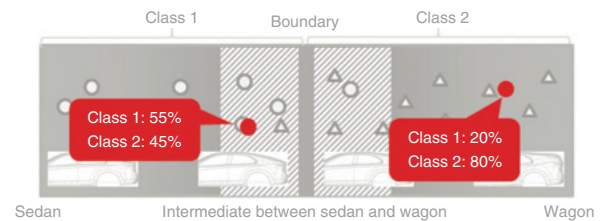


Fig. 12 Boundary treatment of real-time aerodynamic design system

In this system, in order to increase the accuracy of the approximation models, aerodynamic performance is predicted using a method (known as the probability method) in which the linear weighted sum of all the approximation models is determined using the probability of class membership output by SLR. Through the use of the probability of class membership, the probability method is able to linearly interpolate the boundary line domain in each class.

5. Extraction of Feature Values

The real-time aerodynamic design system uses feature values that represent body shapes as numerical parameters as input values for approximation modeling and categorizing. Position coordinates, surface angles, and curvature on the surface of the body are extracted as feature values. The extraction method is discussed in detail below.

5.1. Overview of Method

To make it possible for the real-time aerodynamic design system to treat any arbitrary body shape, it is necessary to be able to stably extract feature values at the desired position irrespective of body shape. A feature value extraction technology that is robust in relation to changes in body shape was developed by conducting system modifications with dynamic programming (DP) matching⁽¹⁰⁾⁻⁽¹²⁾ as a base. DP matching is a method of matching the elements of two different time series patterns, and is widely employed, for example in voice recognition. Figure 13 shows an example of DP matching. The two-dimensional graph to the left of the figure is a reference pattern prepared in advance, and the two-dimensional graph at the bottom of the figure is a reference pattern for the subject of matching. The color contour map in the center of the figure shows absolute values for the difference in distance between the elements of the reference pattern and the subject of matching.

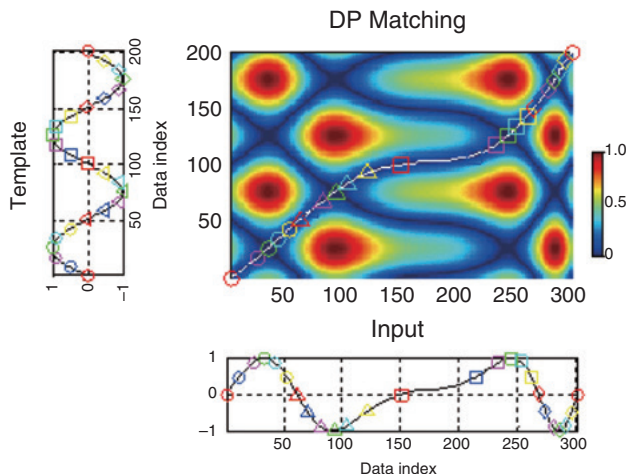


Fig. 13 DP matching method

In order to minimize the error between elements, DP matching determines the optimum path for matching while nonlinearly expanding and contracting the reference pattern. From multiple reference patterns, the pattern with the shortest cumulative distance for optimum paths is selected as the pattern most closely resembling the input pattern. Two major features can be indicated for DP matching: (1) It is able to absorb nonlinear shape changes in the input pattern even in the event that there are few reference patterns; and (2) It is able to automatically determine feature points even for unknown input patterns. With regard to computation time also, the use of DP matching makes it possible to break up optimization problems into partial problems for solution, meaning that computations can be conducted efficiently, with a reduced volume of operations.

To apply DP matching in the feature value extraction section of the real-time aerodynamic design system, two-dimensional cross-sections are extracted from three-dimensional shapes. In order to increase accuracy, one-dimensional complex feature vectors that consider the angle with a tangent to surface in addition to the distance from the center of gravity of the extracted cross-section lines are generated, and DP matching is applied.

5.2. Computation Procedures

Figure 14 shows the DP matching procedures. As the figure shows, feature values are extracted by means of the following four procedures:

- (1) Two-dimensional cross-section lines for the body are extracted from three-dimensional shape data;

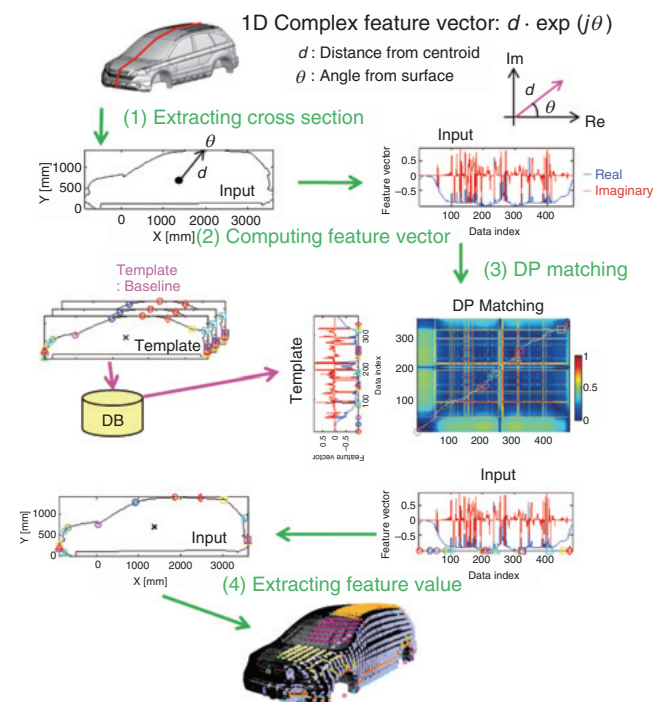


Fig. 14 Procedures of DP matching

- (2) The two-dimensional cross-section lines are converted to one-dimensional complex feature vectors;
- (3) Matching is conducted with the template in the database, and feature values are extracted; and
- (4) With the feature points determined in (3) as a benchmark, feature values are extracted from the entire vehicle body in the form of a grid.

6. Sensitivity Analysis

The correlation between shape changes and aerodynamic performance is extremely important for the user. A variety of methods have been proposed for sensitivity analysis⁽¹³⁾. These methods can be broadly divided into two types: Global methods, which determine sensitivity from a range of data; and local methods, which calculate the gradient with the shape being focused on. While global methods are able to consider large deformations, local methods, as represented by adjoint⁽¹⁴⁾, are only able to respond to small deformations.

In order to compute global sensitivity data at high speeds, the real-time aerodynamic design system uses an online sensitivity analysis system. This system has direct access to the data stored in the database from previous computations and automatic accuracy increase, and is thus able to realize high-speed computation. Spearman coefficients⁽¹⁵⁾ are used as the coefficients of correlation.

6.1. Computation of Degree of Displacement

The degree of displacement is computed by calculating displacement for unstructured triangular data for two vehicle surfaces. The degree of displacement of each vertex is defined as

$$\delta_{i,j} = (v_j - v_i) \circ n_i. \quad (5)$$

Here, δ , v , and n are the displacement, apex, and normal vectors, respectively. The subscripts i and j are the shape at the beginning of displacement and the shape at the conclusion of displacement, respectively. The vertex of the shape at the conclusion of displacement is selected as the point of minimum distance from the apex of the shape at the beginning of displacement.

6.2. Online Sensitivity Analysis

Figure 15 shows the computation procedures involved in online sensitivity processing. The orange squares in the figures show processing, while the gray squares show results and data. As the figure shows, the aerodynamic performance values output from the feature values extracted from the body shape and the approximation models form the input for online sensitivity analysis. The details of each processing operation are presented below.

(1) Find best matching category

The category to which the body shape input by the user belongs is determined from feature values. For

determination of the category, after the Euclidean distance is found from the category baseline data and the input body shape feature values, the data is assigned to the category in which the Euclidean distance is the lowest. The Euclidean distance is calculated using the following equation:

$$d(f^{new}, f^{base}) = \sqrt{\sum_n |f_i^{new} - f_i^{base}| / |f_i^{new} + f_i^{base}|}. \quad (6)$$

Here, f^{new} and f^{base} are the shape input by the user and the feature values of the baseline data in the category, respectively.

(2) Approximate displacements

Determination of the degree of displacement between the surface shapes in the design input by the user and the data classified in the same category is necessary for sensitivity analysis. Several thousand or more data exist in a single standard category, and an extremely long computation time is needed in order to determine all displacements. A method of significantly reducing computation time by rough estimation of the degree of displacement was therefore developed. First, the displacement of the data input by the user in relation to the baseline data in the category, $\delta^{n,b}$, is calculated. With $\delta^{n,b}$ as an offset value, the level of displacement against the remaining shapes in the category is estimated. The following equation is used for the calculation of the level of displacement of the shape data input by the user, n , and the data in the same category.

$$\tilde{\delta}^{n,m} = \delta^{n,b} + \delta^{b,m} + e^{n,m}. \quad (7)$$

Here, $\delta^{n,b}$ is the value calculated for the level of displacement against the baseline using Eq. (6), $\delta^{b,m}$ is the level of displacement extracted from the database, and $e^{n,m}$ is error.

Figure 16 shows the results of displacement calculations. The color contours on the left show displacement as calculated using Eq. (6), and those on the right show displacement as calculated using eq. (7). The error for displacement is 1.8%.

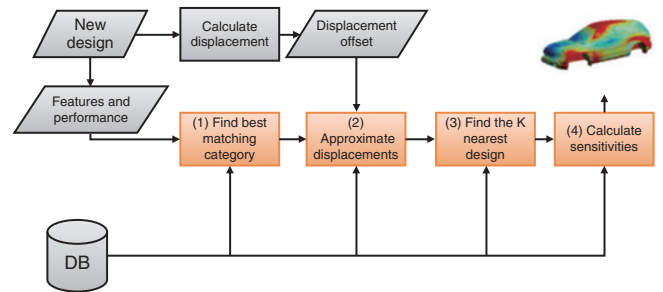


Fig. 15 Flow chart of the online sensitivity approximation

(3) Find the K-nearest designs

K-nearest neighbor (KNN) was introduced to the system in order to determine similar shapes to the shape input by the user. KNN searches for similar data from the level of displacement rather than searching for similar designs from the feature values of the body. The value of K , which expresses the number of similar shapes, is determined empirically, and is set with consideration of the calculation time and the scope of the search.

(4) Sensitivity calculation

Spearman correlation coefficients are calculated based on the number of similar shapes, K , the level of displacement of the respective shapes, $\tilde{\delta}^n$, and the aerodynamic performance values. The sensitivity, ρ_i^n , of the apex v_i of shape n input by the user is calculated based on linear approximation of the level of displacement and the displacement component of the aerodynamic performance values. Figure 17 shows the results of sensitivity analysis. The output results show the direction of increase or decrease in aerodynamic performance values. Positive values indicate an increase in aerodynamic performance values when the shape is deformed in the direction of the normal vectors of the body surface.

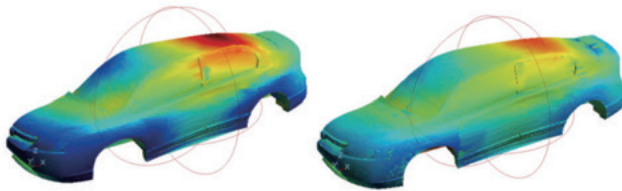


Fig. 16 Qualitative comparison between the calculated displacements on the left and the estimated displacements on the right. The displacement values are projected onto a color value between minimum (blue) and maximum (red) displacement value

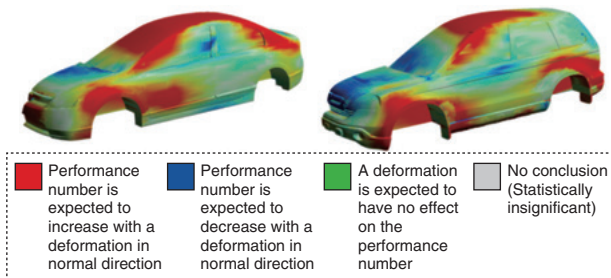


Fig. 17 Visualization of the sensitivity information, calculated concerning the Civic coupe model (left) and the CRV model (right)

7. System Performance

The performance of the real-time aerodynamic design system was evaluated. Figure 18 shows an outline of the evaluation method. First, data for the generation of approximation models and data for performance evaluation were generated using the procedure discussed in Section 2.1.1. Following this, performance values were predicted for the data for performance evaluation using the generated approximation models, and the performance of the approximation models was evaluated. 15942 cases of data for the generation of approximation models were generated from five baselines, and 1201 cases of test data for evaluation were generated from five baselines. The evaluation of the performance of the approximation models used the indexes discussed below.

(1) Correct rate, τ

The ratio of cases in which magnitude relationships for the performance of two randomly extracted designs is correctly judged was used for evaluation. In concrete terms, the number of successes and failures in predicting tendencies for all pairs that can be drawn from the data for performance evaluation was determined, and the cumulative ratio was used as the correct rate. This method of evaluation is generally known as the Kendall rank correlation coefficient. The correct rate, τ , is expressed in the following form:

$$\tau = N_{cp} / \frac{1}{2} n(n-1) \quad (8)$$

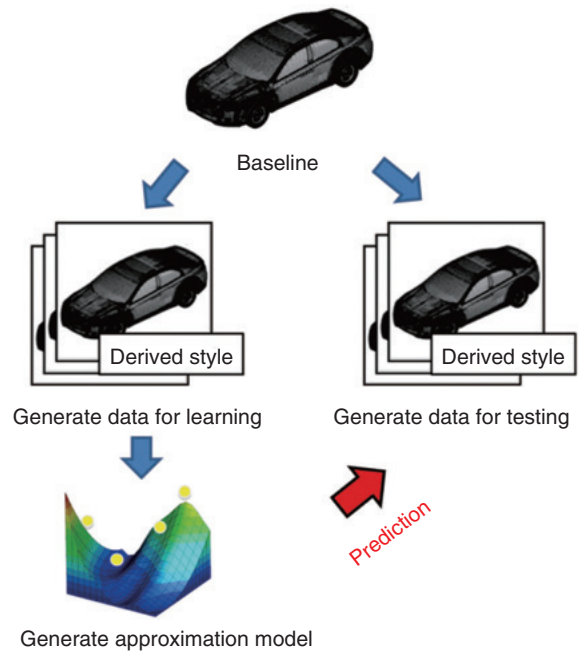


Fig. 18 Procedure of evaluation for real-time aerodynamic design system

Here, N_{cp} represents the number of pairs that successfully predicted tendency.

(2) Correlation coefficient, r_p

The correlation coefficient, r_p , is an index that expresses the correlation between two data sets (predicted values for data for evaluation and measured values), and is expressed as:

$$r_p = \frac{\sum_{n=1}^N (y_n - \bar{y})(\hat{y}_n - \bar{\hat{y}})}{\sqrt{\sum_{n=1}^N (y_n - \bar{y})^2} \sqrt{\sum_{n=1}^N (\hat{y}_n - \bar{\hat{y}})^2}} \quad (9)$$

Here, y is the measured performance value (CFD calculation value) for the data for evaluation, made up of N data, and $\hat{y}$ is the predicted value. $\bar{y}$ is the average value of y , and $\bar{\hat{y}}$ is the average value of $\hat{y}$. The closer the correlation coefficient to 1, the stronger the correlation and the more accurate the reproduction by the model. The closer the correlation coefficient to 0, the weaker the correlation and the less accurate the reproduction by the model.

(3) Root mean square error (RMSE)

RMSE is calculated using Eq. (10). The higher the value, the greater the prediction error.

$$RMSE = \sqrt{\sum_{n=1}^N (y_n - \hat{y}_n)^2 / N} \quad (10)$$

System performance was evaluated using the evaluation indexes discussed above. The approximation models achieved the following performance values:

- Correct rate: 91%
- Correlation coefficient: 0.95
- RMSE: 1.1%

In addition, excepting display time and certain other parameters, the system's time for prediction of aerodynamic performance was within several seconds.

These results indicate the effectiveness of the real-time aerodynamic design system from the perspectives of accuracy and computation time.

8. Conclusion

A real-time aerodynamic design system able to calculate aerodynamic performance and shape sensitivity in real time was developed. The developed system offers the following features:

- High-speed calculation through use of approximation models
- High-accuracy calculation of multiple approximation models by means of a categorizing technology
- Online calculation of sensitivity
- Automatic accuracy increase function

An evaluation of the system produced the following results:

- Correct rate: 91%
- Correlation coefficient: 0.95
- RMSE: 1.1%
- Calculation time (excepting display section and others): Within several seconds

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