

Prediction of Tooth Root Bending Stress in Hypoid Gears for Drivetrain

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ABSTRACT

A technology for the prediction of tooth root bending stress that considers changes in the gear meshing position has been developed to help enable accurate evaluation of the tooth root strength of hypoid gears used in drivetrains. The gear sets in actual vehicles are supported by a number of other components, and deformation of these components in operation produces different gear meshing positions to those observed when the gears are unloaded. These changes in meshing position increase the stress on the tooth roots. A hypoid gear tooth root stress simulation that considers the stiffness of the supporting components was therefore developed. This technology displayed a good correlation with measurement results for stress distribution along the tooth bottom in the direction of the tooth width and for maximum stress values.

Strength evaluations based on tooth root stress obtained in this way also increased the accuracy of prediction of the lifespans of hypoid gears.

1. Introduction

From considerations of environmental protection, the reduction of weight is today demanded in automotive development in order to reduce CO₂ emissions and increase fuel efficiency. At the same time, the transmission torque load on automotive drivetrain parts is increasing, as a result of increases in engine power and response to market needs (for example, in the manufacture of SUVs). In the area of 4WD, high-performance 4WD systems are being developed to boost the driving stability, extending to systems other than those whose main purpose is handling on snowy roads, and the frequency of operation of these systems during vehicle use is also increasing. Against this background, the ability to accurately predict loads on individual parts in order to realize designs that balance low weight with strength is also becoming increasingly important in 4WD development.

This paper will focus on the hypoid gears that determine the drivetrain framework of 4WD systems.

The contact position of the tooth surfaces of hypoid

gears changes under torque load. In addition, because the thickness of the teeth is not constant in the tooth-width direction, the stress generated on the tooth roots fluctuates. The magnitude of the change in meshing position varies with the configuration, arrangement, and shape of the parts that support the gears, and with the amount of transmitted torque, among other factors. The accurate prediction of the stress generated on the tooth roots by these changes in meshing position is an important factor in evaluating the bending strength of hypoid gear tooth roots in actual vehicles.

A method of analyzing bending stress on individual gears has previously been reported as research on tooth root stress in hypoid gears⁽¹⁾.

The Gleason Corporation software^{(2), (3)} generally employed to calculate tooth root bending stress in hypoid gears is an effective tool in conducting calculations for individual gears, but does not enable calculation of the stiffness of the gear supports.

A prediction technology was therefore developed that, using structural analysis, helps to enable the realization of a seamless process from reproduction of the degree of

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change in the meshing position in a real vehicle to the calculation of the bending stress on the tooth roots. The use of this method has increased the accuracy of calculation of tooth root bending stress in hypoid gears.

2. Development Goals

In a torque-splitting 4WD system using a standard FF vehicle as a base, engine torque is transmitted to the rear axle from the transmission positioned on the front axle and the transfer unit attached to its final reduction stage via the propeller shaft and the rear differential unit (Fig. 1).

The function of the transfer unit is to transfer the torque and the rotation split from the final reduction stage in a right angular direction via the hypoid gears, and to output it to the propeller shaft.

The transfer unit has a significant effect on the packaging layout of the engine housing. Given the relationship of positioning between the transfer unit and chassis parts (such as the steering gearbox) and exhaust system parts, if the transfer unit is too large, it may produce a change in the vehicle height or the positioning of the engine. Because of this, it is desirable to reduce the size of the transfer unit.

The use of a cassette method to achieve separation from the transmission and two-stage multiplication using helical and hypoid gears can be identified as specific characteristics of Honda transfer units.

The advantage of the Honda system is that by concentrating functions in the transfer unit, a unified transmission framework can be employed, without consideration of the differences between 2WD and 4WD. In addition, the use of two-stage multiplication offers freedom in the positioning of the transfer unit.

However, as Fig. 2 shows, in addition to reaction force from the hypoid gears, the transfer unit housing is subject to reaction force from the helical driven gears positioned at the exterior of the bearing support points. In addition, because the transfer unit housing is made up of housing

A, which supports the ring gears, and housing B, which supports the pinion gears, a minute degree of slip occurs at the junctions. This slip is also a factor in altering the meshing position of the gears and increasing tooth root bending stress.

It is necessary to achieve sufficient strength in the hypoid gears even under these conditions. Because no method of accurately predicting changes in the meshing position has been available, gear strength has conventionally been evaluated based on experience. This made it necessary to develop a method of analyzing tooth root bending stress capable of reproducing the state of gear-meshing in actual operation.

Given this, development goals were formulated as follows:

- (1) The development of a technology for the prediction of hypoid gear tooth root bending stress that considers the state of meshing in actual operation; and
- (2) The achievement of increased accuracy in predicting the lifespan of hypoid gears.

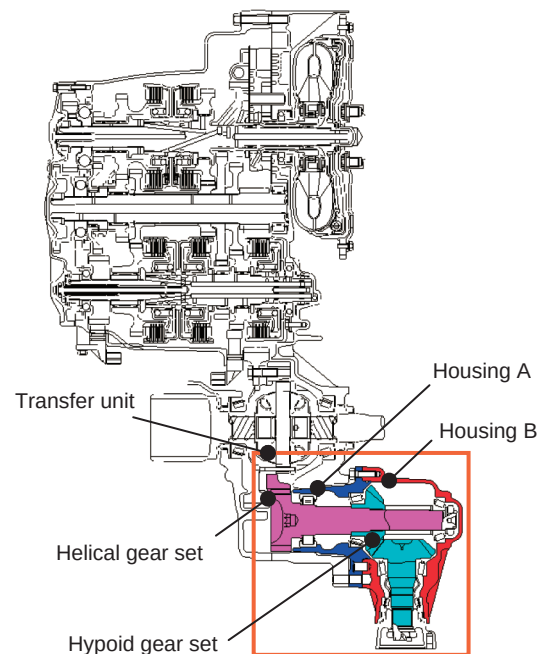


Fig. 2 4WD main section

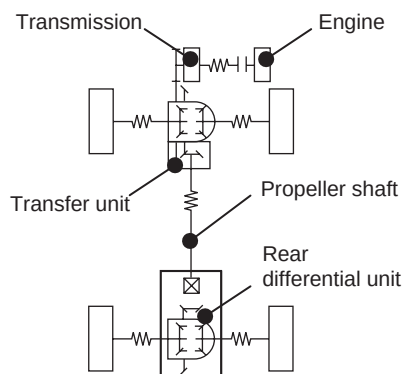


Fig. 1 4WD vehicle model

3. Technology for the Prediction of Hypoid Gear Tooth Root Bending Stress

In order to reproduce the state of meshing in the transfer unit during actual operation, a structural analysis model was formulated that incorporated the stiffness, fitting, and other characteristics of the helical gears, shafts, and hypoid gear set that form the torque transmission path; and of the housing, bearings, and other parts that support them.

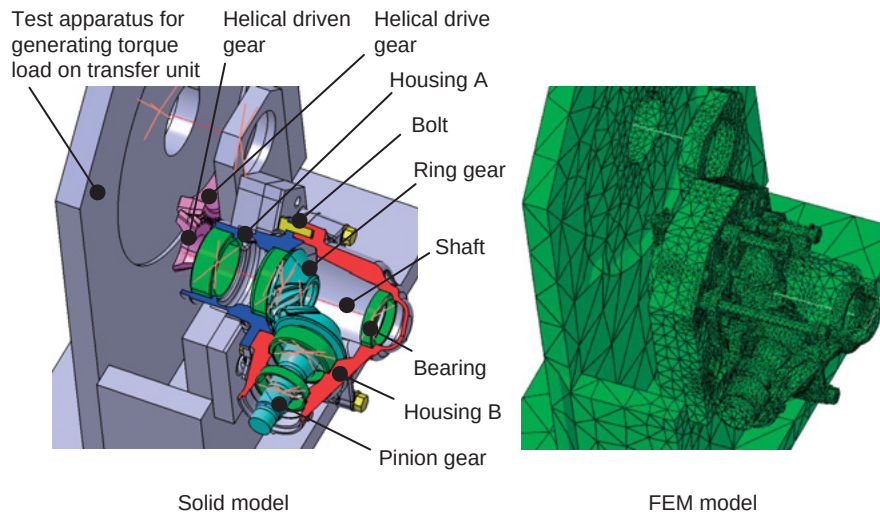


Fig. 3 Components of transfer unit for analysis

Because the formulation of a simulation method was the main focus of the project, the hypoid gear tooth surfaces were modeled after measurement of their actual form using an optical non-contact measurement device⁽⁴⁾.

3.1. Simulation Model

Figure 3 shows the configuration of the transfer unit assembly model used in the simulation of hypoid gear tooth root bending stress.

In order to take into consideration the deformation and the minute degree of slip of the connected surfaces of the housing, as discussed above, a coefficient of friction was defined as a condition for the connected surfaces of the housing, and clearances were set for the fitted sections. In addition, shaft diameter, materials, and axial force were set as parameters for the coupling bolts⁽⁵⁾.

The next part of this section will discuss the modeling of the internal structural parts.

Because the bearings are subject to reaction forces in both radial and thrust directions, it is important to set appropriate stiffness values in relation to both forms of reaction force. Given this, the inner surface of the outer ring was connected to the central point of the bearing

using a rigid bar, and this central point was connected to the shaft, and values were set for the stiffness of the bearings in the radial, axial, and inclined directions (Fig. 4). In addition, defining the external face of the outer ring and the housing as contact surfaces helped to enable the reproduction of the compressive force acting in the direction of load on the bearings and the resulting separation in the opposite direction⁽⁶⁾.

An elastic bar was used for the shaft axle, and the diameter and materials of the axle were defined in order to reproduce deflection and torsion. Finally, the hypoid gears were connected to the shaft axle using a rigid bar, and their fitting with the spline gears was reproduced.

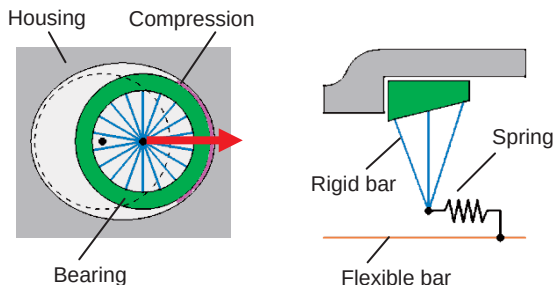


Fig. 4 Bearing model



Fig. 5 3D model making process

3.2. 3D Modeling of Hypoid Gears

Figure 5 shows the flow of gear modeling.

Reverse engineering software was used to generate polygon data in which the tooth surfaces and tooth bases were integrated based on the point group data obtained from the optical non-contact measurement device. At this stage, the polygon data also reproduced the concavities and convexities at the level of tooth surface roughness, including marks left by the tooth cutting and abrasion processes. If this data was employed in the structural analysis, the number of meshes would increase significantly, making it necessary to remove this detailed form data. A section of the measured shape within a range of 5 μm was therefore isolated from the polygon data, and smooth surface data that expressed the form of the grooves on the teeth was generated.

Finally, the gear model was completed by removing this surface form from a solid model of the base material form created from a diagram.

Next, an FEM model was formulated based on this solid model. To help enable the reproduction of torque transmission by gear meshing in the structural analysis, contact surfaces were defined between the ring gear and pinion gear tooth surfaces, and a coefficient of friction was set. Secondary elements were selected to help enable the curved surface form to be traced to the maximum extent possible. A mesh size of 0.4 mm or less was employed for the tooth bases, since it would affect the accuracy of the simulated stress. In addition, in order to reduce the mesh number, six teeth were used, with all teeth that would not affect the results of the analysis being omitted from the model.

3.3. Analysis Procedure

Figure 6 shows the analysis procedure used in this method.

In Step 1, the state of the transfer unit immediately following fitting in a jig was reproduced. First, the tightening axial force on the bolts was input and the housings connected, and the preset load on the bearings was input. At this stage, the hypoid gears and the helical gears were constrained to help prevent rotary motion.

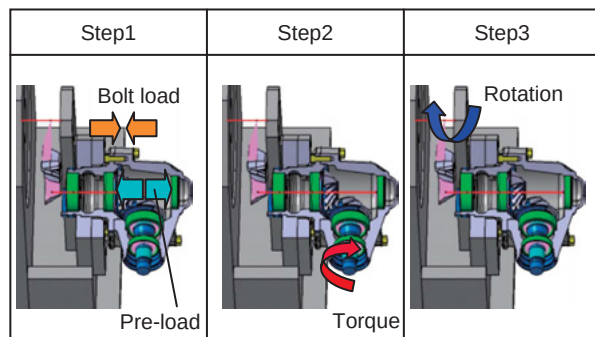


Fig. 6 Analysis procedure

In Step 2, the torque was loaded on the spline section of the pinion gears fitted to the propeller shaft, in order to reproduce the state of the static torque load.

In Step 3, forced rotation was applied to the helical drive gears at the side of the jig in the opposite direction to torque input in order to reproduce the state of meshing during hypoid gear drive. Here, the tooth root bending stress was simulated at 0.5° increments of the angle of rotation of the helical drive gears from the commencement to the completion of hypoid gear meshing.

Finally, the rest of this section will discuss the method employed to process the results.

Tensile stress is generated on the tooth roots of the meshing teeth as shown in Fig. 7.

Figure 8 shows stress contours for three meshing phases from the commencement to the completion of meshing, and examples of the results of calculation of the distribution of tooth root bending stress in the direction

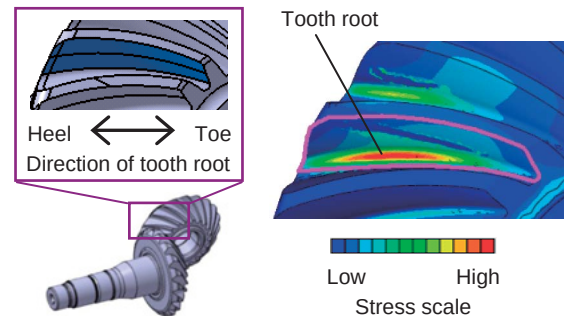


Fig. 7 Tooth root bending stress

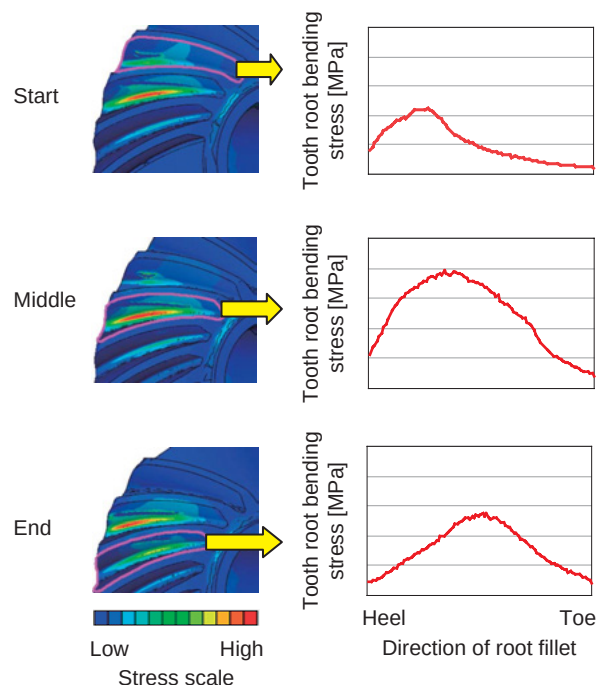


Fig. 8 Tooth root bending stress in different phase

of tooth width. As these examples show, tooth root bending stress varies depending on the phase of meshing. Since strength must be evaluated at the maximum level of stress generated, the results were processed to find the highest stress values in the meshing phases from the commencement to the completion of meshing.

3.4. Simulation Results

The distribution of tooth root bending stress was compared when the stiffness of the gear supports was considered and when it was not considered (Fig. 9) in order to determine the effect of the degree of change in the meshing position on tooth root bending stress. All the support components were defined as rigid bodies in the analysis model that did not consider the stiffness of the supports.

When the stiffness of the supports was considered, the position at which the peak stress was generated shifted in the direction of the heel, and the peak stress value increased. At the same time, the stress in the direction of the toe decreased.

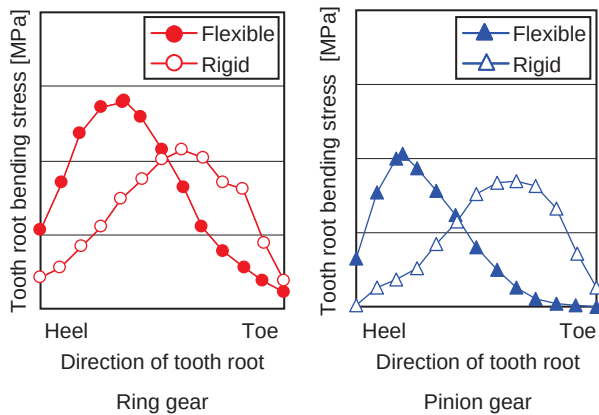


Fig. 9 Influence of stiffness of gear supports

4. Verification of Accuracy of Analysis

4.1. Measurement of Tooth Root Bending Stress

In order to verify the accuracy of the analysis, the tensile stress on the tooth roots was measured with the gears meshed under a static load in the bench-top torsion tester shown in Fig. 10.

The torsion tester has a configuration in which, as in the simulation model, torque is applied from above the pinion gear shaft, and rotation is held steady via the helical gear.

As Fig. 11 shows, for the measurements five strain gauges were attached to one gear tooth. Tooth root bending stress was measured for three gears, with the position of the gauges staggered in the direction of tooth width, giving stress values for 15 points in the direction

of tooth width. The stress values used for verification were, as in the case of the simulation model, the highest values in the phases of meshing of the hypoid gears from commencement to completion.

In past durability tests using actual gears, the hypoid gear set used in the measurements had displayed a tendency for the ring gears to fracture first. In the measurements discussed here also, stress values were highest in the ring gears. Given this background, the ring gears were focused on in the verification process.

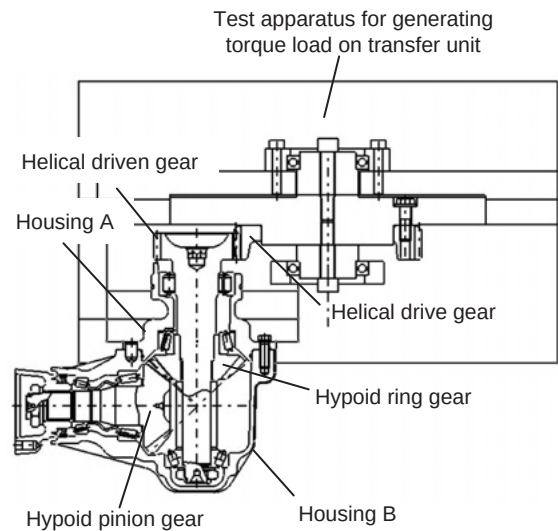
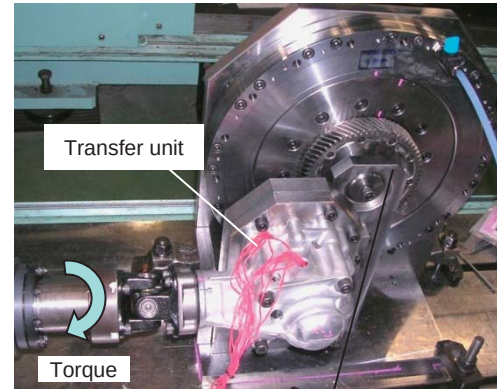


Fig. 10 Torsion tester

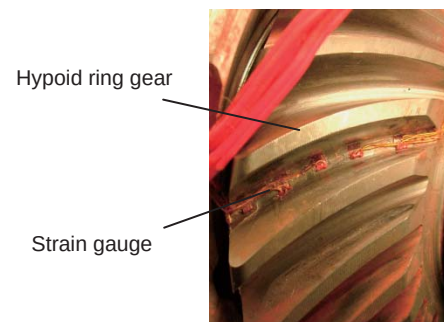


Fig. 11 Gear stress measurement set-up

4.2. Comparison of Calculated Values and Measured Values

Figure 12 shows a comparison of stress distribution for the calculated values and the measured values. A good correlation was obtained between the distributions. In the actual vehicle, the position at which peak stress was

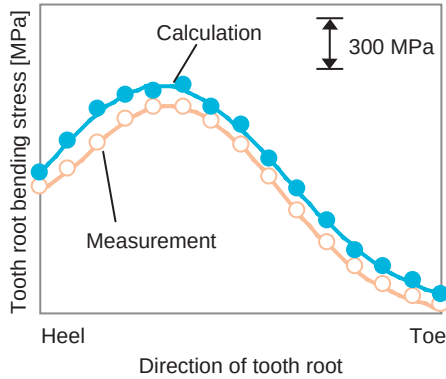


Fig. 12 Distribution of tooth root bending stress for ring gear

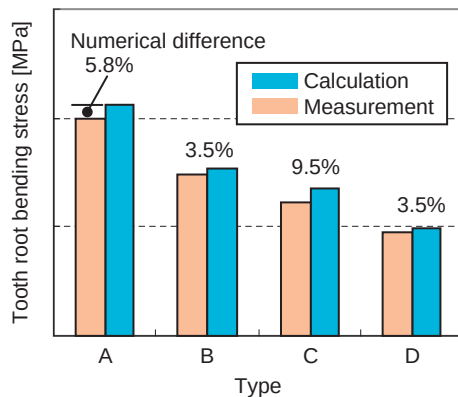


Fig. 13 Comparison of actual measurements and analysis values for tooth root bending stress

generated shifted towards the heel, and the load on the tooth root was not distributed across the entire tooth width. This shows, as in the case of the above-mentioned simulation model that considered the stiffness of the supports, that the degree of change in the meshing position of the gears affects the bending stress on the tooth roots.

Next, the accuracy of the peak values is discussed. Figure 13 shows a comparison between the calculated values and values measured in four hypoid gear sets using different gear sizes. Values were lowest for gear size A, and highest for gear size D. The simulation values tended to be high in comparison to the measured values. This is thought to be due to the fact that the mesh width in the simulation model was low in relation to the width measured by each strain gauge. The difference between the figures is 10% or less.

5. Evaluation of Hypoid Gear Tooth Root Bending Strength

Fatigue tests were conducted on hypoid gears in order to determine the correlation between stress values in the simulation and the number of meshing cycles until fracture.

5.1. Hypoid Gear Fatigue Test

In order to formulate a life diagram, four types of transfer unit were created by assembling combinations of three different gear types of differing sizes and four types of housing of different shapes, and durability tests were conducted (Fig. 14). The materials and carburizing conditions were identical for all the hypoid gears.

Figure 15 shows the durability tester, which was able to apply a continuous high load. The tester detected the vibration generated when a gear broke using a G sensor, and was set to stop automatically when this occurred. In

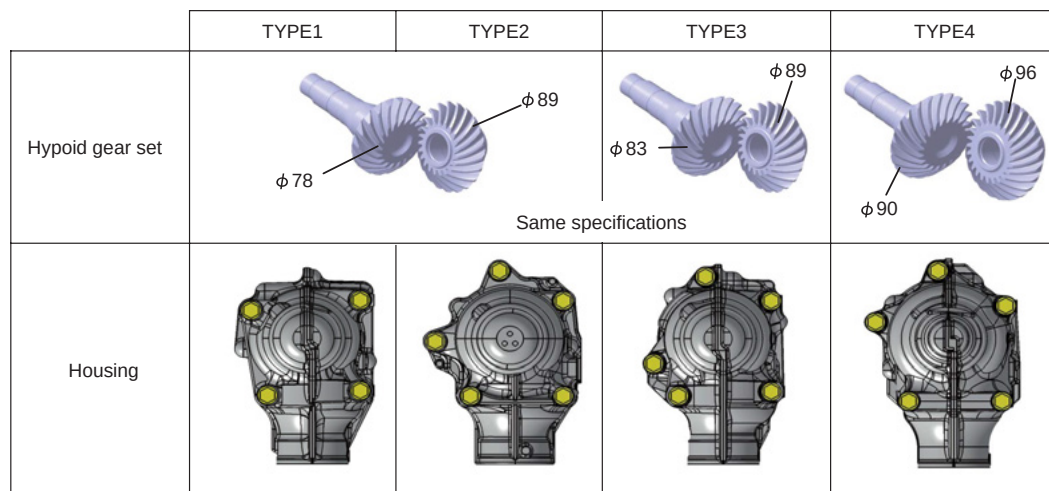


Fig. 14 Combination of hypoid gear sets used for examination and housing

the tests, torque was input from the side of the pinion gears and the unit was operated until a hypoid gear broke. The number of meshing cycles that had elapsed up to that point was then determined.

As operating conditions, input torque values of 500, 750, 900, and 1100 Nm were employed, and the number of input rotations was constant.

Figure 16 shows a gear after the completion of the

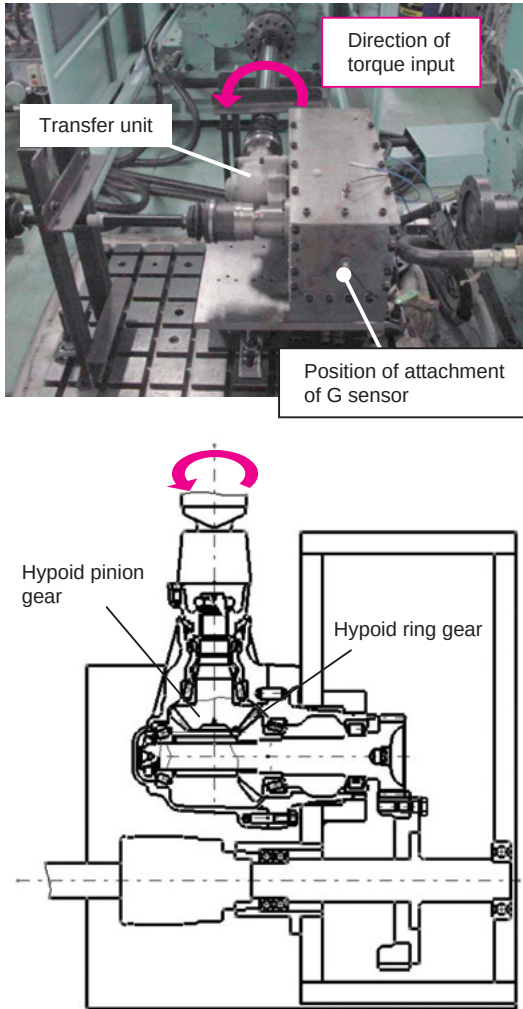


Fig. 15 Equipment used for durability test



Fig. 16 Status of gear damaged in durability test

test. The tooth broke away from the tooth root in the direction of the heel, where tooth root stress is high.

5.2. Relationship between Tooth Root Bending Stress and Durability Life

Figure 17 shows a plot of the gear fatigue test results, with simulation stress values, S , on the vertical axis, and the number of meshing cycles at the point of breakage in the fatigue tests, N , on the horizontal axis.

The coefficient of correlation between stress values obtained from the simulation and gear fatigue life was 0.94. This indicates that the method helps to enable accurate prediction of gear life, and that the size of hypoid gears can be set without relying on experience.

In addition, the usefulness of the method will not be limited to the optimal design of gear size, but can also be used in relation to parameters such as the shape of the housing and the arrangement of the bearings when the transfer unit framework is under consideration at the initial stage of design.

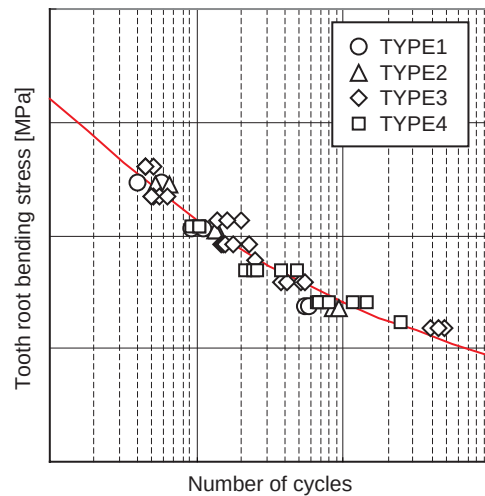


Fig. 17 Stress number diagram

6. Conclusion

- (1) A hypoid gear tooth root bending stress simulation was developed that reproduces the stiffness of the gear supports and the state of gear-meshing, in order to accurately predict tooth root bending stress in drivetrain hypoid gears.
- (2) Simulation stress values predicted measured values to an accuracy of within 10%.
- (3) A high correlation was obtained between simulation stress values and the number of meshing cycles at breakage, indicating that the method helps to enable accurate prediction of fatigue life.

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