

Calibration of Tooth Profile Measurement and Uncertainty Analysis Based on Ball Artifact Method

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ABSTRACT

Tooth profile measurements were calibrated using a double ball artifact that is traceable to a national standard of length. From the calibrated results, an evaluation technology that quantifies the uncertainty of tooth profile measurement in a 1-to-1 correspondence with a tooth profile of arbitrary specifications was established. In this evaluation technology, the work of calibrating the measuring machine has been minimized, and measurement simulations have been made feasible by modeling the statistical properties, as well as algebraic calculation from the model during tooth profile measurement, enabling the reliability of tooth profile measurement to be statistically evaluated. As a result, during tooth profile inspections on the manufacturing floor, practical inspection accuracy is now guaranteed.

1. Introduction

Many of the functions of a gear, such as quietness, rely on the accuracy of the 3-dimensional curved form.

For example, there is a report⁽¹⁾ to the effect that the profile accuracy for satisfying quiet driving is approximately 1 μm , and advances are being made concerning gear manufacturing accuracy. Gear measuring machines are widely used to measure and inspect gear tooth accuracy. In order to cope with the increasingly high level of gear accuracy, however, highly accurate measuring technology and a high level of calibration technology for measuring the appropriateness and reliability of measuring accuracy are being demanded. For example, 1 μm is demanded for class 0 (highest) accuracy under ISO 1328-1 and JIS B 1702-1 gear accuracy standards.

Previously, in order to calibrate a gear measuring machine, a highly machined and evaluated gear was used as a calibration standard (hereafter called “artifact”). However, when using a gear artifact and aiming for high accuracy as a standard thereto, the following issues exist at present.

- (1) There is a limit to the level of accuracy to which a gear can be machined.

- (2) There is no method of measuring a gear to high accuracy and guaranteeing it.

Also, when a gear is used as an artifact, an artifact consisting of a single specification gear is not very appropriate for enabling an accuracy guarantee to be rigorously performed on gears of numerous specifications, as the object of inspection. Consequently, to establish a method that can cope with the increasing accuracy of gears and practically guarantee the inspection accuracy in the inspection on the production line is not an easy task. In light of this situation, in recent years a method of calibration for tooth profile measurement using a double ball artifact (DBA) has been proposed in ISO/TR 10064-5. In the double ball artifact method, a steel ball having a high degree of sphericity and highly accurate surface properties, and is treated as a pseudo-involute gear, is measured, and the deviation between the measurement results and the ideal profile is quantified. Here, the deviation is defined as the tooth profile measurement deviation.

In addition to the conventional purpose of DBA use, the calibration method reported in this paper aims to quantify multiple kinematic errors from the calibration data, such as the positioning errors of the guide and rotary table, which constitute the kinematic mechanism of the

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Table 1 Specification of DBA

Name	Material	Thermal expansion coefficient [$1/K$]	Size [mm]
Base plate	Clearceram-Z	0.8×10^{-7}	$\phi 130$
Steel ball	Bearing steel	11.6×10^{-6}	S $\phi 25.4$

gear measuring machine, and determine the extent to which the kinematic error contributes to the tooth profile measurement deviation. This is because the kinematic error of the mechanism has systematic repetition, and it is expected that the accuracy of the tooth profile measurement deviation can be enhanced by numerically correcting the systematic error. In addition, reference was made to the tooth profile measurement deviation, which was quantified by calibration, and evaluation technology for quantifying the uncertainty of measurement in a 1-to-1 correspondence with a tooth profile of arbitrary specifications was established.

As a result of this evaluation technology, the difference in the measurement accuracy prior to performing a profile inspection can be quantitatively predicted, using gear measuring machines that have mutually different performance, and a traceability system, which is being viewed with importance in recent years, has been constructed. Because DBAs are calibrated by a national measuring organization, a gear measuring machine that is calibrated by a DBA is also traceable to a national standard. In this paper, the method of quantifying the uncertainty of measurement of the abovementioned tooth profile of arbitrary specifications is described in the sequence below.

- (1) Making a geometrical model of measurement using DBAs, and making a formula for obtaining a solution
- (2) Uncertainty analysis of the tooth profile measurement deviation obtained by calibration

- (3) Method of tooth profile measurement uncertainty simulation using the results of uncertainty analysis of tooth profile measurement deviation

2. Principle of Calibration Using a Double Ball Artifact

2.1. Design of a Double Ball Artifact

Figure 1 shows a schematic diagram of a DBA. In the datum plane of the DBA, the two balls are located as shown in Fig. 1(b). During measurement, balls A and B are considered to be a shaft center perpendicular to the datum plane, and an involute gear profile, respectively, and are made to function accordingly. For this reason, the distance between the centers of balls A and B should be determined with high accuracy.

Table 1 shows the basic specifications of the fabricated DBA. The base plate uses extremely low expansion material Clearceram to minimize the thermal deformation of the center distance between A and B, which depends on the measurement environment. Balls are used because precision machining is easy, a highly accurate profile can be easily obtained, and calibration that is traceable to a national standard has been realized.

2.2. Making a Model of the Principle of Calibration, and Method of Obtaining a Solution

Figure 2 shows a schematic diagram of a model obtained when a DBA is installed on a gear measuring

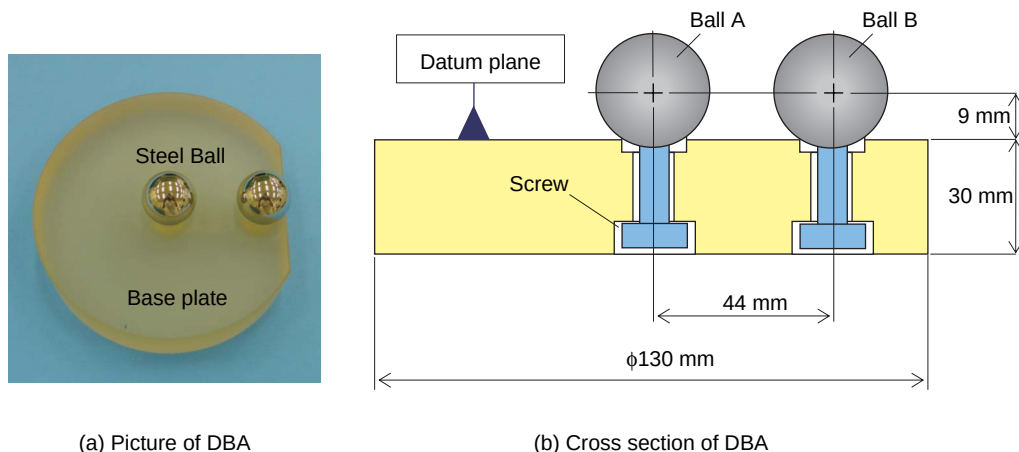


Fig. 1 Schematic of produced DBA

machine, and Fig. 3 is a photograph of the actual layout. The layout is such that the center position of the rotary table of the gear measuring machine coincides with the position of the center o of ball A of the DBA, and minute layout errors are numerically corrected. Based on this layout, when the center of ball B moves from position a , where it used to be, prior to the start of the measurement, to position c with rolling angle ε , the probe scans line l_1 , which touches the base circle radius r_b , parallel to the y axis. However, in actual fact, the x axis has a positioning error e_x , so the probe moves along line l_2 .

At this time, the y coordinate of the probe position of which the rolling angle ε is a function is expressed by Eq. (1), where the y coordinate of center point c of ball B is y_c , and the y component from point c to the center position of the probe touching ball B is ΔY .

In the case where it is assumed that there is no error in the positioning of the measuring machine and the form of the probe, and the probe scans line l_1 while touching the ideal involute, if the radius of the probe is r_p , the y coordinate of the center of the probe is

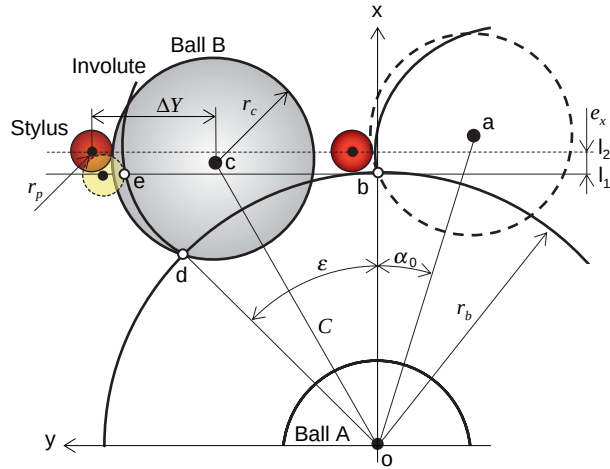


Fig. 2 Schematic model of DBA alignment on gear measuring machine

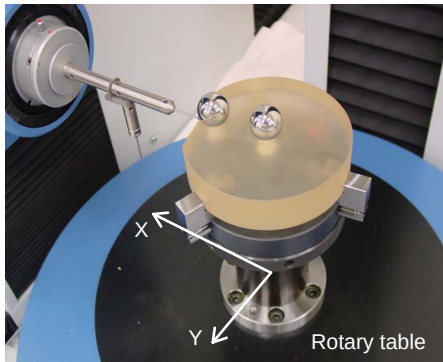


Fig. 3 Picture of DBA alignment on gear measuring machine

expressed by Eq. (2).

From Eqs. (1) and (2), the difference Δf between Y_s and Y_l is defined as the deviation between the arc of the ball and the involute form, as shown in Eq. (3).

$$\begin{aligned} Y_s &= f(\varepsilon) \\ &= Y_c + \Delta Y \\ Y_c &= C \sin(\varepsilon - \alpha_0), \\ \Delta Y &= \sqrt{(r_c + r_p)^2 - (r_b + e_x - C \cos(\varepsilon - \alpha_0))^2} \end{aligned} \quad (1)$$

Where,

C : Master ball center distance
 α_0 : Initial angle of rotary table
 ε : Rolling angle
 e_x : x axis positioning error
 r_c : Master ball radius
 r_p : Probe radius
 r_b : Base circle radius

$$Y_l = r_b \cdot \varepsilon + r_p \quad (2)$$

$$\Delta f = Y_s - Y_l \quad (3)$$

In this model, the initial angle of the rotary table is used as a parameter, hence compared to the Kondo model⁽²⁾, it is stated simply, and is advantageous in that the parameter estimation calculation process and uncertainty analysis can be simplified.

In Eq. (1), the initial angle α_0 and x axis positioning error e_x have errors with respect to a nominal value determined from the design value of the mechanism, hence they are used as parameters that perform calibration simultaneously. The theoretical value vector and the observed value vector of the form deviation of Eq. (3) acquired with respect to the continuously changing rolling angle ε are Δf_{Theo} and Δf_{obs} , respectively. If the difference between these vectors is defined as vector Y_{obs} in Eq. (4), it will be feasible to arrange simultaneous equations concerning the unknown parameter p to be estimated. If the Jacobian matrix comprising the partial differential coefficient of p in Eq. (4) is defined as $A^{(3)}$, a design value will be assigned to p as the initial value, and a convergence value of p will be obtained from the non-linear least squares method (Gauss-Newton method), which updates the value as p^+ according to Eq. (5). Here, the suffix t of the matrix indicates a transposition, and S indicates the error matrix of Y_{obs} . In this analysis, the error propagation to the parameter estimation of the measurement is not taken into account, so S is a unit matrix.

$$Y_{obs} = \Delta f_{obs} - \Delta f_{Theo}, Y_{obs} = f(p), p = \begin{pmatrix} \alpha_0 \\ e_x \end{pmatrix} \quad (4)$$

$$p^+ = (A^t S^{-1} A)^{-1} A^t S^{-1} Y_{obs}, p = p + p^+ \quad (5)$$

Table 2 Setting condition for DBA measurement parameters

Parameter	Set value	Estimated value	Unit
Module	7.7639	—	mm
Number of teeth	12	—	—
Radius of base circle	43.7741	—	mm
α_0	16.63802	16.63799	deg
e_x	0	0.4	μm

In the parameter that is to be evaluated as a kinematic error in the gear measuring machine, there exists a systematic positioning error e_y of the y axis. Here, in order to calibrate $\mathbf{p}$ and e_y simultaneously, e_y was modeled according to Eq. (6) from an approximation polynomial, using the rolling angle ϵ , which is based on the rotary table. In Eq. (6), $\mathbf{B}$ and $\boldsymbol{\beta}$ are a basis function matrix, based on a Legendre function⁽⁴⁾, and its coefficient vector, respectively.

At this time, the observed value $\mathbf{Y}_{ey}$ consisting of $\mathbf{Y}_{obs}$ from which the effect of estimation value $\mathbf{p}$ has been removed is equivalent to the positioning error on the y axis, so this relationship is shown in Eq. (7). Note that e_y can be estimated using the least squares method from Eq. (7).

$$e_y = \mathbf{B}\boldsymbol{\beta}, \mathbf{B} = (B_0(\epsilon) \cdots B_n(\epsilon)), \boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \vdots \\ \beta_n \end{pmatrix} \quad (6)$$

$$\begin{aligned} \mathbf{Y}_{ey} &= \mathbf{Y}_{obs} - \mathbf{A}\mathbf{p} \\ &= \mathbf{B}\boldsymbol{\beta} \end{aligned} \quad (7)$$

2.3. Result of Estimating the Tooth Profile Measurement Deviation and Measuring Machine Kinematic Error

To acquire data for the purpose of estimating the tooth profile measurement deviation, a DBA having Table 1

specifications was used to set the initial values used in the gear specifications and unknown parameter estimation $\mathbf{p}$, as shown in Table 2. Figure 4 shows the theoretical curve Δf_{Theo} and measured curve Δf_{obs} in the settings of Table 2. In the results of Fig. 4, data obtained by suitably translating Δf_{obs} so that it coincides with Δf_{Theo} is used, and the result of calculating $\mathbf{p}$ from Eq. (5) is as shown in Table 2. It can be confirmed that there is a minute error in the positioning of the x axis and the rotary table. Also, the estimated value of e_y , calculated from Eq. (7), is as shown in Fig. 5, and the observed data $\mathbf{Y}_{ey}$ is approximated with good accuracy.

With this method, multiple kinematic errors, such as the translation and rotating components of the mechanism of the measuring machine, can be simultaneously estimated from a single observed result, and it is also easy to statistically describe the effect that the error $\mathbf{S}$ in the observed value in Eq. (5) has on the reliability of the calculated kinematic error. On the other hand, the construction of a model and the calculation process tends to become complicated.

Compared to this method, the conventional method⁽⁵⁾, which successively estimates each kinematic error value from multiple measurement methods permits a model to be simply constructed; however, the cost of each kinematic error measurement is high and explaining the statistical mutual relationship of the estimated kinematic

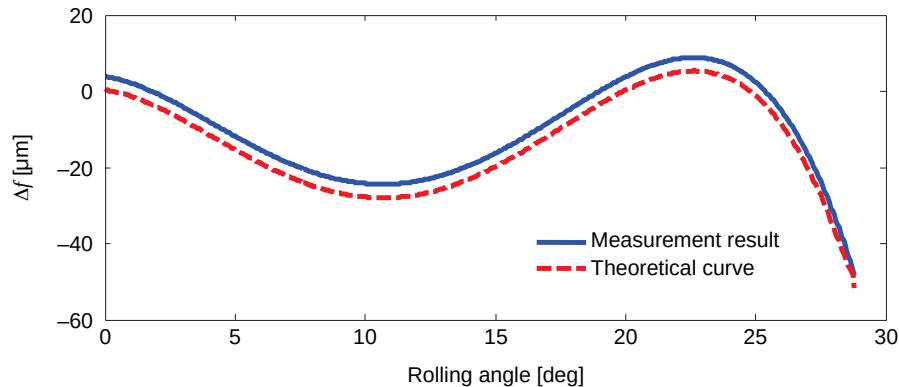


Fig. 4 Example of calibration measurement results

errors from multiple measurement data is challenging. The tooth profile measurement deviation means that the modeled parameters p and e_y simultaneously contribute to the measurement, so Fig. 6, which indicates the simultaneous contribution of p and e_y , is defined as the tooth profile measurement deviation f_{ref} . If e_y and f_{ref} in the figure are compared with each other, a difference in the absolute values can be confirmed.

In the calibration using DBA, the kinematic error in the mechanism of the measuring machine was deemed to be estimable, so the tooth profile measurement deviation and its mutual relationship were clarified quantitatively.

3. Quantification of the Uncertainty of the Tooth Profile Measurement Calibration Results

3.1. Double Ball Artifact Uncertainty Evaluation

Using the tooth profile measurement deviation estimated from calibration, the construction of a method of evaluating uncertainty of the tooth profile measurement of a gear that has arbitrary specifications satisfying the following is studied.

- (1) The calibration results should be traceable to a national standard.

- (2) A method that minimizes the necessity for a preparatory test for making an uncertainty evaluation, and uses numerical calculations that can be easily developed on the production line. In this sub-section, the uncertainty that contributes to tooth profile measurement deviation that depends upon the double ball artifact is quantified.

The combined uncertainty of Δf in Eq. (3) is derived according to GUM⁽⁶⁾ guidelines. Here, it is necessary that the combined standard uncertainty of Y_s and Y_l be both known. Because Y_l is a nominal value, only the combined standard uncertainty of Y_s in Eq. (1) is quantified.

The combined uncertainty of Y_s can be expressed as shown in Eq. (8), in which the uncertainty of the random component that depends upon the gear measuring machine is u_M and the uncertainty of the random component that depends upon the DBA is u_{DBA} . The standard uncertainty of parameter p is expressed as $u(p)$.

Here, because r_b is the nominal value of the base circle radius, it is not considered a source of uncertainty. Also, the uncertainty of temperature dependency is not taken into account in U_{DBA} . This is because, even if the temperature change during measurement is assumed to be an excessive value of 1 K with respect to the DBA coefficient of linear expansion, $0.8 \times 10^{-7}/K$, the thermal deformation with respect to the evaluated ball distance of

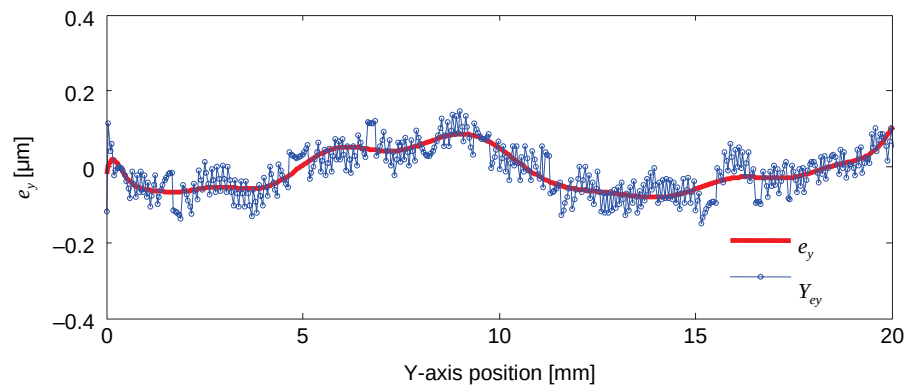


Fig. 5 Estimation result of Y-axis positioning error

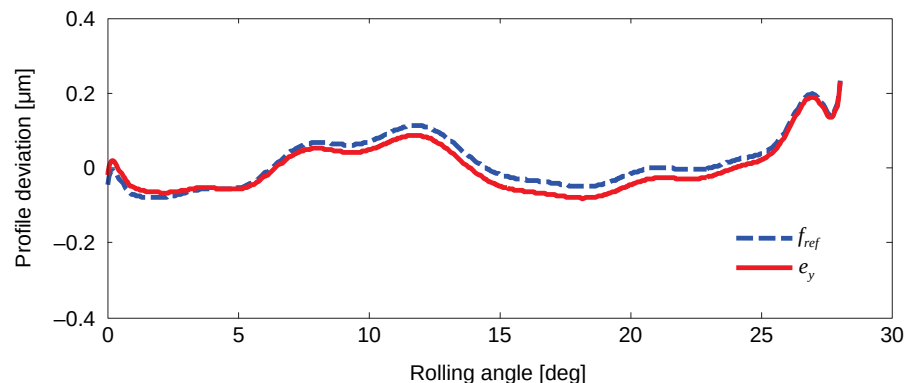


Fig. 6 Estimation result of tooth profile deviation

Table 3 Sources of DBA uncertainty

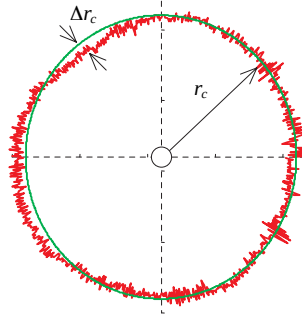
Uncertainty sources	Symbol	Standard uncertainty	Type of distribution	Type
Length between two master spheres	C	0.13 μm	Normal	B
Radius of master sphere	r_c	60 nm	Normal	B
Thermal coefficient of DBA	α_{DBA}	$0.1 \times 10^{-7}/\text{K}$	Rectangular	B
Temperature of DBA during measurement	t_{DBA}	1 K	Rectangular	A

44 mm is $4 \times 10^{-3} \mu\text{m}$, which is so small that it can be ignored.

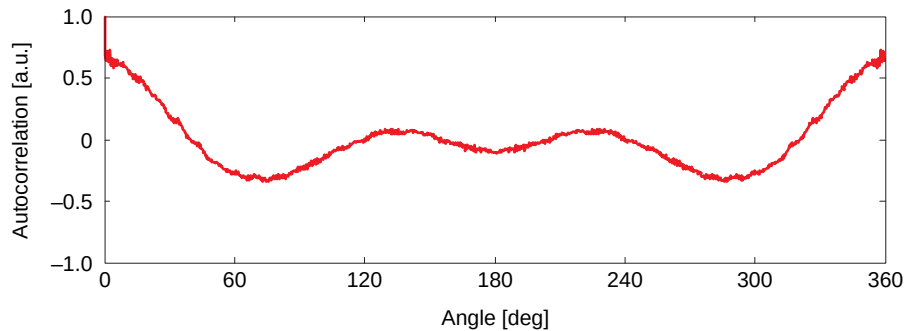
$$\begin{aligned}
 u^2(Y_s) &= u_M^2 + u_{\text{DBA}}^2 \\
 u_M^2 &= \left(\frac{\partial Y_s}{\partial e_x} \right)^2 u^2(e_x) + \left(\frac{\partial Y_s}{\partial e_y} \right)^2 u^2(e_y) + \left(\frac{\partial Y_s}{\partial r_p} \right)^2 u^2(r_p) + \left(\frac{\partial Y_s}{\partial \varepsilon} \right)^2 u^2(\varepsilon) \\
 u_{\text{DBA}}^2 &= \left(\frac{\partial Y_s}{\partial r_c} \right)^2 u^2(r_c) + \left(\frac{\partial Y_s}{\partial C} \right)^2 u^2(C)
 \end{aligned} \quad (8)$$

Concerning Eq. (8), the uncertainty source of DBA and its standard uncertainty are summarized in Table 3. The accuracy of the radius r_c in the red path of the master ball was made 60 nm, based on the results of research⁽⁷⁾ involving measurement of the roundness of a ball of the same size and grade as that of the ball used.

Figure 7(a) shows the form of the ball's red path acquired using a coordinate measuring machine that had a measurement accuracy of $u = 0.6 + L/1000 \mu\text{m}$ and L : Evaluation length mm. Also, if the form data and its residual from the least squares circle with respect to the form data is defined as Δr_c , the autocorrelation with respect to the Δr_c angle becomes that shown in Fig. 7 (b), and periodic correlation with respect to the phase in which it anticipates the machining history can be seen on the surface of the ball. However, because the measuring accuracy of the ball surface form using a coordinate measuring machine is insufficient for an evaluation accuracy of the roundness of a 60 nm ball, and because the existence or otherwise of surface form correlation does not contribute very much to u_{DBA} , correlation is ignored in this analysis.



(a) Form deviation of master ball



(b) Autocorrelation of master ball form deviation

Fig. 7 Form deviation and autocorrelation of master ball on DBA

Taking the foregoing into consideration, when the combined standard uncertainty u_{DBA} ($k = 1$, k : coverage factor) that conforms to Eq. (8) is calculated, the result becomes that shown in Fig. 8(a). From Fig. 8(a), the reliability range due to u_{DBA} of curve Δf is as shown in Fig. 8(b). The maximum contribution to the uncertainty of u_{DBA} corresponds to the center distance of the ball, and the center distance is evaluated by a laser interferometer. Consequently, regarding Δf of Eq. (3), a profile standard that is traceable to the length is realized.

3.2. Evaluation of Uncertainty of Gear Measuring Machine

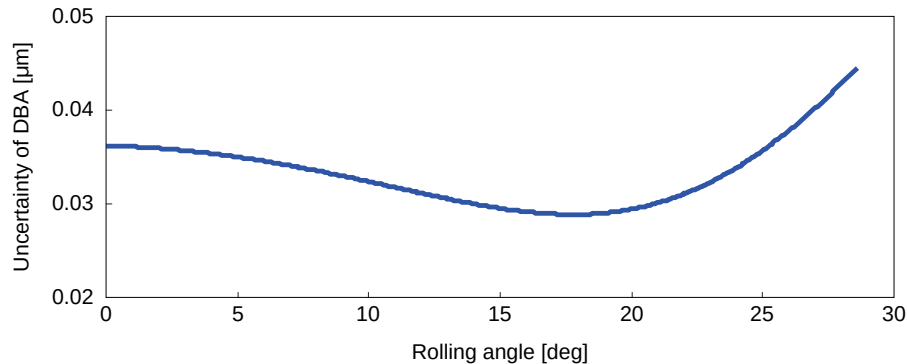
The uncertainty sources that are dependent on the gear measuring machine are e_x , e_y , r_p , and ε from the tooth profile measurement model in Eq. (1). If the standard uncertainty in these parameters can be determined, the uncertainty of tooth profile measurement by the measuring machine can be evaluated from Eq. (8) based on the error propagation rule.

As an evaluation method that does not rely on the error propagation rule, a method that estimates the final measurement uncertainty by means of simulation that simultaneously generates suitable random numbers for each of the model uncertainty sources is also being

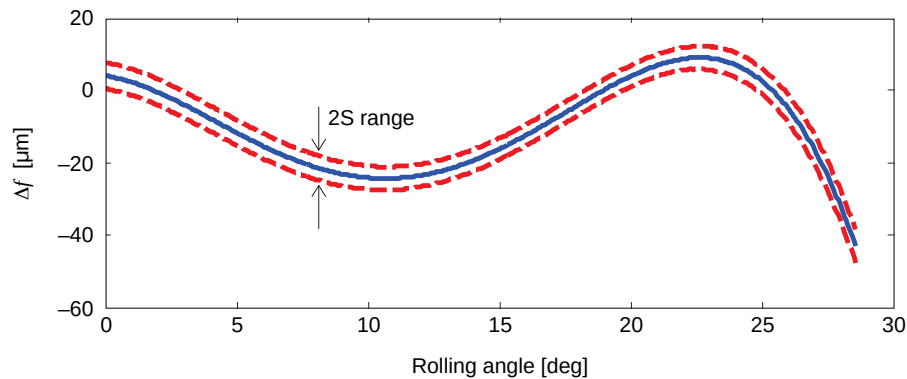
developed⁽⁸⁾.

Regarding the abovementioned simulation method, it is necessary to evaluate such uncertainty sources as the positioning error of the guide and the positioning error of the rotary table in advance, by means of an external measuring machine such as a laser interferometer, and there are cases in which the uncertainty of the evaluation machine is added to the uncertainty of each factor, resulting in an excessively high evaluation. Also, it is impractical for the user to perform this type of evaluation work because of cost limitations.

Here, because simple, repetitive measurement of DBAs can be performed in a short time, the uncertainty acquired from DBAs is estimated as the uncertainty item u_M of the random component that depends upon the gear measuring machine in Eq. (8). u_M has a variation that depends upon the rolling angle as shown in Fig. 9, and has an average uncertainty $u_{M(Ave)}$ of 0.035 μm . Because the absolute value of u_M is minute, and because it has no marked systematicity with respect to phase, DBA uncertainty, u_{DBA} , and the combined standard uncertainty of u_M , that is, the uncertainty u_{com} of the tooth profile measurement deviation, are constant regardless of the angle of rotation, and are defined in Eq. (9). Here, u_{com} is an uncertainty of approximately 0.05 μm ($k = 1$) as shown in Fig. 9.



(a) Uncertainty of DBA depending on rolling angle



(b) Theoretical curve and uncertainty resulting from DBA

Fig. 8 Uncertainty of DBA estimated from calibration certification of NMIJ (National Metrology Institute of Japan)

$$u_{com} = \sqrt{u_{M(Ave)}^2 + u_{DBA}^2} \quad (9)$$

3.3. Monte Carlo Simulation of Tooth Profile Measurement Deviation

Using the uncertainty u_{com} of the quantified tooth profile measurement deviation, the reproduction by means of a Monte Carlo simulation of the repetitiveness with respect to the tooth profile measurement deviation f_{ref} estimated as shown in Fig. 6 has been studied. An outline of the principle of the simulation is as shown in Fig. 10, and consists of the following items. In this sub-section, stages (2) to (4) are formulated.

- (1) The tooth profile measurement deviation f_{ref} estimated from measurement performed using a DBA is taken as a reference value.
- (2) A uncertainty matrix of the tooth profile measurement deviation that bears a 1-to-1 correspondence to the phase of the reference value f_{ref} is constructed.
- (3) Measurement random errors are generated by singular

value decomposition of the uncertainty matrix.

- (4) Data of the tooth profile measurement deviation obtained by combining the reference value f_{ref} and the random measurement errors is reproduced.

In the case where the number of data components of the reference value f_{ref} is defined as n , if the $n \times n$ unit matrix is defined as $\mathbf{I}$, the calibration uncertainty matrix Σ is expressed as Eq. (10) from Eqs. (8) and (9).

If $\mathbf{U}$ and $\mathbf{V}$ are defined as orthogonal matrices, the result of a singular value decomposition of the uncertainty matrix Σ becomes as shown in Eq. (11). The diagonal elements of matrix $\mathbf{D}$ are the square of the respective singular values s , and have properties that correspond to the uncertainty of the mutually uncorrelated normal distributions.

If each column element of matrix $\mathbf{V}$ is defined as $\mathbf{V}_i$ ($i: 1$ to n), one set of measurement random errors in a 1-to-1 correspondence with the phase of the reference value f_{ref} can be generated by performing one trial of Eq. (12). Here, the Monte Carlo simulation indicates the process

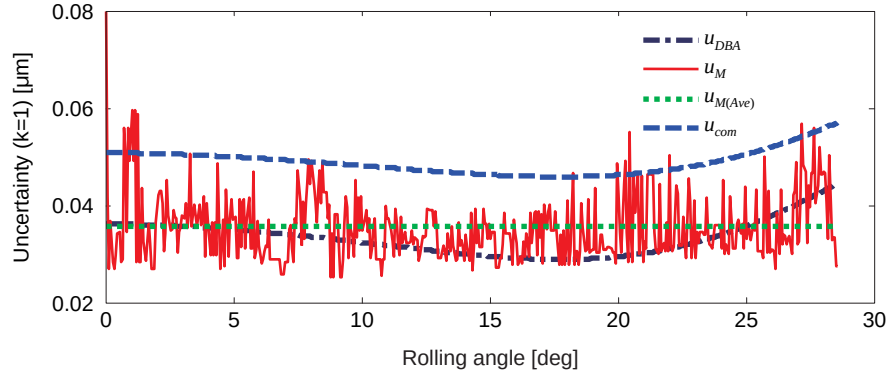


Fig. 9 Combined standard uncertainty of tooth profile measurement

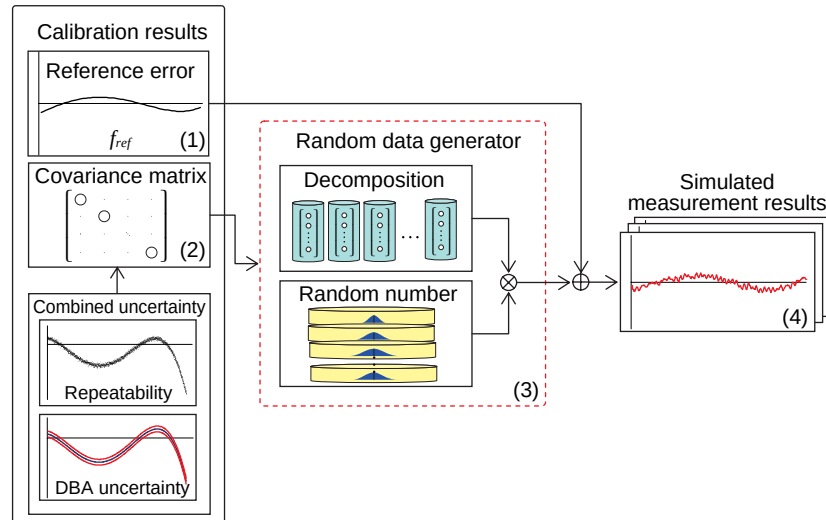


Fig. 10 Statistic simulation method for evaluating uncertainty of tooth profile calibration

that generates normal random numbers at standard deviation 1 independently, for each probability variable $\mathbf{X} = (x_1 \dots x_n)$ of the expected value of zero. Note that there is no limit to the number of simulation trials.

$$\begin{aligned} \Sigma &= (u_M^2 + u_{DBA}^2) \mathbf{I} \\ &\approx u_{com}^2 \mathbf{I} \end{aligned} \quad (10)$$

$$\Sigma = \mathbf{U} \mathbf{D} \mathbf{V}^t, \mathbf{D} = \begin{pmatrix} s_1^2 & & \\ & \ddots & \\ & & s_n^2 \end{pmatrix} \quad (11)$$

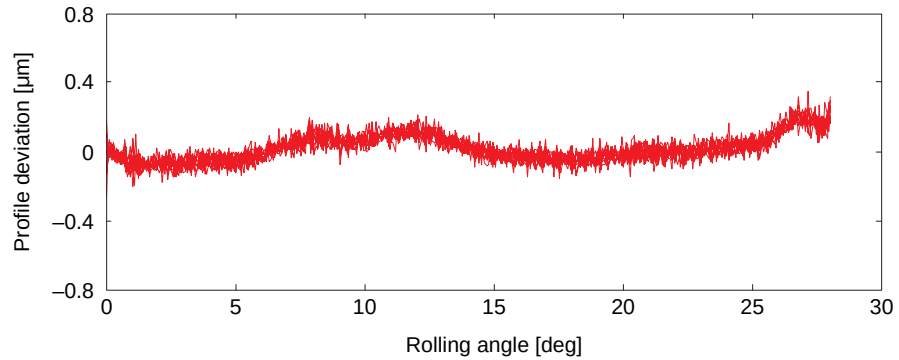
$$f_{ran} = s_1 \cdot x_1 \mathbf{V}_1 + s_2 \cdot x_2 \mathbf{V}_2 + \dots + s_n \cdot x_n \mathbf{V}_n, \mathbf{V} = (\mathbf{V}_1 \mathbf{V}_2 \dots \mathbf{V}_n) \quad (12)$$

As an example of a simulation trial, Fig. 11 shows 10 sets of pseudo tooth profile measurement deviation data generated by combination due to 10 trials of f_{ran} and reference value f_{ref} . Compared to the results of 10 cycles of measurement shown in Fig. 11(a), the pseudo tooth profile measurement deviation data of Fig. 11(b) also exhibits a tendency to be systematic, however, because DBA uncertainty is added, the behavior of the random component is large, and all of the uncertainty of the uncertainty sources occurs simultaneously.

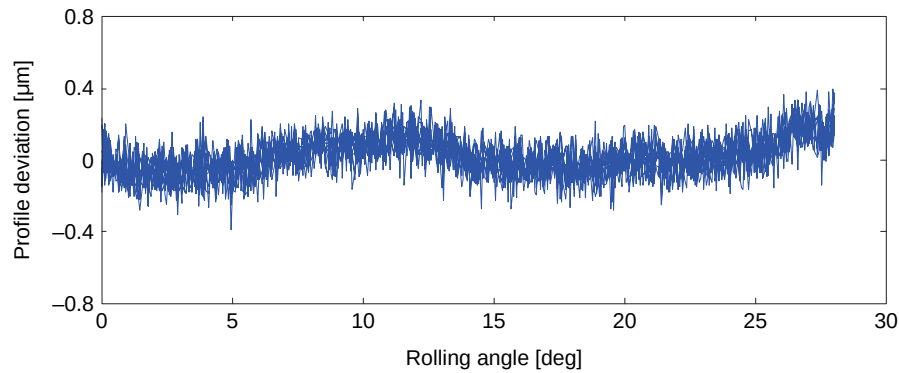
The uncertainty of the evaluation of the DBA ball distance used was $0.13 \mu\text{m}$ ($k = 1$), however it has been reported⁽⁷⁾ that when publicly known technology is used, a ball distance of $0.01 \mu\text{m}$ ($k = 1$) is achieved. If the ball distance is evaluated at $0.01 \mu\text{m}$, it is feasible to estimate that the uncertainty of the pseudo tooth profile measurement deviation of Fig. 11(b) is several percent on the low side.

4. Tooth Profile Measurement Uncertainty Analysis

The tooth profile measurement simulation results and the tooth profile measurement with respect to arbitrary specifications that can be estimated from these results are converted into uncertainty values. The minimum to maximum shift range y_{min} to y_{max} of guide y, evaluated by calibration of the measuring machine from the DBA specification of Table 3, extends from approximately 0 to 25 mm. If module m used for the gear is made $0.5 \leq m \leq 3$ mm in steps of 0.5, for example, and the pitch circle diameter D_0 is made $20 \leq D_0 \leq 120$ mm in steps of 20, the shift range of the y axis needed for performing these inspections becomes as shown in Fig. 12. The range of gear specifications that satisfies y_{max} is $D_0 \leq 80$ mm and



(a) Profile deviation repeatability observed by DBA



(b) Profile deviation repeatability generated by simulation

Fig. 11 Comparison of tooth profile deviation between observation and simulation

$m \leq 2.5$ mm. Consequently, here the uncertainty of gear measurement that simultaneously satisfies $20 \leq D_0 \leq 80$ mm and $0.5 \leq m \leq 3.0$ mm is quantified.

When the combination of an arbitrary m and D_0 is determined, the rolling angle range ε_{range} and shift range Y_{range} of the guide y necessary for scanning the probe from the dedendum to the addendum are determined as Eqs. (13) and (14), and the shift range of the guide y equivalent to the inspection range of the profile conforming to JIS B1702-1 and ISO 1328-1 is determined.

$$\varepsilon_{range} = \sqrt{\left(\frac{r_t}{r_b}\right)^2 - 1}, r_b = r_0 \cos(\alpha) \quad (13)$$

Where,

r_b : Base circle radius

r_t : Addendum circle radius

r_0 : Pitch circle diameter

α : Pressure angle

$$Y_{range} = r_b \cdot \varepsilon_{range} \quad (14)$$

Figure 13 shows the results of referring to the tooth profile measurement deviation f_{ref} , which corresponds to the shift range of guide y , and performing a tooth profile measurement simulation. Here, five trials were performed, and for each simulation trial, the least squares straight line of the mean profile and parameters of the profile deviation, the total profile deviation F_α , profile form deviation $f_{f\alpha}$, and the profile angle deviation $f_{H\alpha}$, all of which were standardized by ISO 1328-1, were calculated, then the standard deviation corresponding to the number of trials was calculated. The uncertainty is as shown in Eq. (15) where the average value of the total profile deviation, for example, is $\bar{F}_\alpha$ and the standard deviation is $\sigma(F_\alpha)$.

$$u(F_\alpha) = \bar{F}_\alpha + 2\sigma(F_\alpha) \quad (k=2) \quad (15)$$

As an example of the analysis results, Fig. 14 indicates F_α , $f_{f\alpha}$ and $f_{H\alpha}$. It can be confirmed that F_α and $f_{f\alpha}$ have linearity with respect to the pitch circle diameter, module, and uncertainty.

The merits of this uncertainty evaluation, based on tooth profile measurement calibration and calibration simulation, are as follows.

- (1) The necessary preparatory test and calibration work for evaluating the uncertainty of the measuring machine were minimized, and it was feasible to simulate tooth profile measurement by utilizing an algebraic model of the statistical nature of measurement in the tooth profile measurement deviation evaluation, and by calculation, thus eliminating the work of statistical evaluation by measurement work.
- (2) The necessary accuracy with respect to a profile of arbitrary specifications could be determined in advance, by simulating tooth profile measurement, and the uncertainty for each inspection result could also be guaranteed.
- (3) The DBA uncertainty is taken into consideration in the

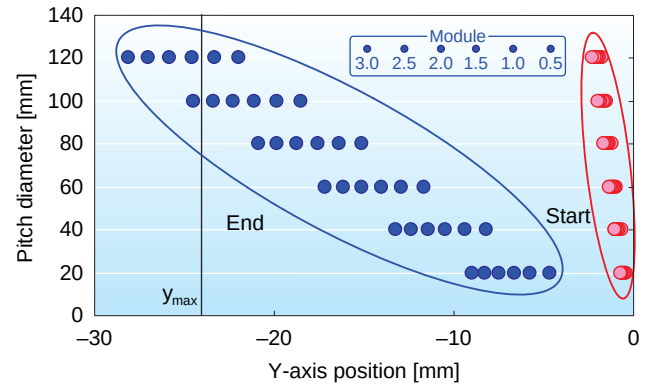


Fig. 12 Range of Y-axis movement for tooth profile measurements

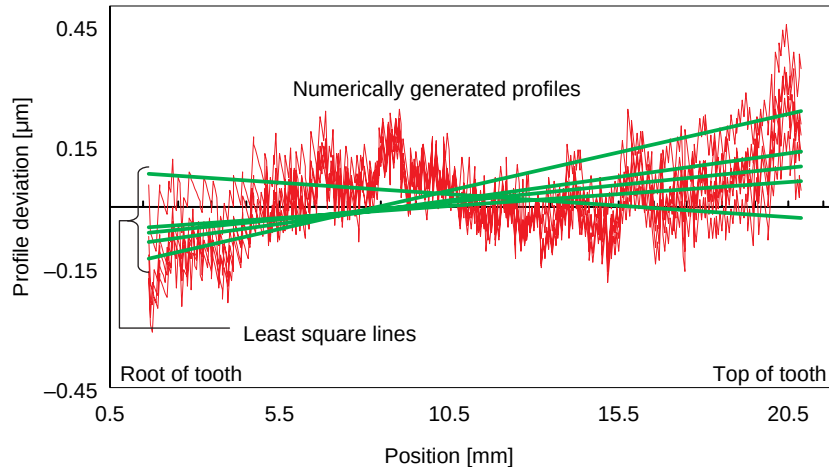
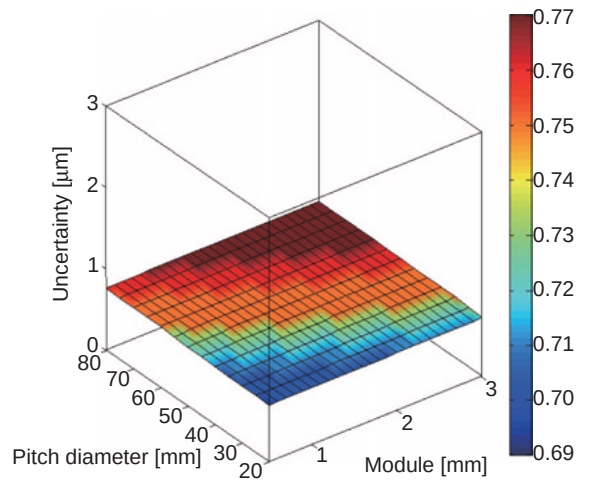
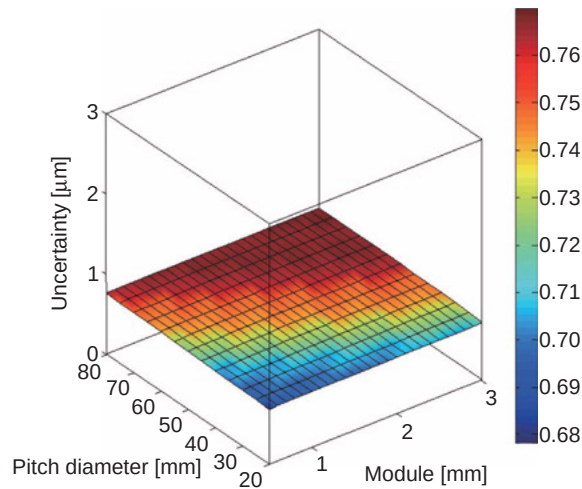
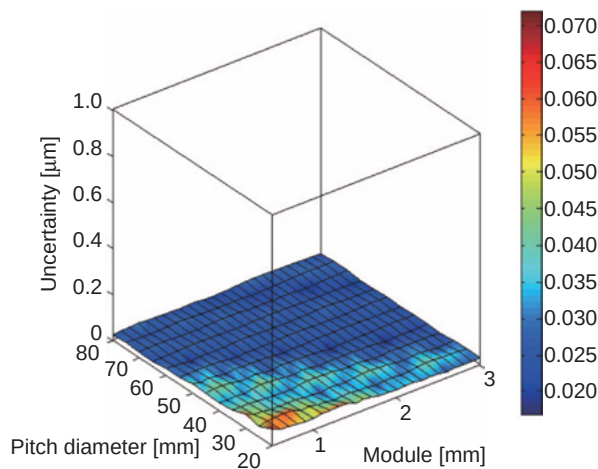


Fig. 13 Generated tooth profile deviation and average tooth profiles by simulation

(a) Uncertainty evaluation for F_α (b) Uncertainty evaluation for $f_{f\alpha}$ (c) Uncertainty evaluation for $f_{H\alpha}$ Fig. 14 Examples of uncertainty evaluation ($k=2$)

uncertainty of the analyzed profile deviation parameter, which is traceable to a national standard, so the difference in accuracy between multiple measuring machines is uniformly evaluated.

5. Conclusion

Calibration of tooth profile measurement was performed using DBAs. From the calibrated results, evaluation technology that quantifies the uncertainty of tooth profile measurement in a 1-to-1 correspondence with a tooth profile of arbitrary specifications was established. The conclusions obtained are as follows.

- (1) Modeling of calibration using DBAs and a method of estimating the kinematic error in the mechanism of a measuring machine were formulated. The quantitative mutual relationship between the profile measurement deviation and kinematic error were clarified by means of this method.
- (2) The uncertainty of tooth profile measurement was algebraically modeled, and singular value decomposition was used to formulate a method of simulating a tooth profile measurement that has traceability, from the computed results.
- (3) From the simulation, a method of evaluating the uncertainty of the tooth profile deviation parameter in the measurement of a tooth profile of arbitrary specifications was derived.

References

- (1) Kubo, A., Komori, M.: Kyoukitansin, No. 3 p. 1-2 (2004) (in Japanese)
- (2) Kondo, K.: A new proposal of primary artifact for calibration of gear measuring machines and calibration method, The Japan Society of Mechanical Engineers Bibliography, Vol. 8, p. 508-515 (2005)
- (3) Nakagawa, T., Koyanagi, Y.: Saisho Jijyou Hou niyoru Jikken Deeta Kaiseki, Toukyou Daigaku Shuppan, p. 35 (1982) (in Japanese)
- (4) Shinozaki, H., Matsushita, Y.: Ouyou Suuchi Keisanhou Nyuumon (Jyou), Korona Sha, p. 175 (1976) (in Japanese)
- (5) Zhang, G.: A Displacement Method for Machine Geometry Calibration, Annals of the CIRP, Vol. 37, No. 1, p. 515-517 (1998)
- (6) BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML: Guide to the expression of uncertainty in measurement, ISO, First edition (1993)
- (7) Kondo, Y., Noguchi, S., Kondo, K., Osawa, S., Naoi, K., Takatsuji, T., Kurosawa, T.: Tooth Form Evaluation Using Ball Artifact: Development of a Measuring Instrument of Ball Center Distance Traceable to National Standard of Length, Journal of the Japan Society for Precision Engineering, Vol. 71, No. 10, p. 1260-1265 (2005)

- (8) Tanaka, H., Shimojo, M., Kajitani, M., Sato, T., Takeda, R., Kojima, M., Yuzaki, M., Kato, S., Shibuya, Y., Kondo, T.: Evaluation of measurement uncertainty in gear measuring instruments by the Monte Carlo simulation, The American Society for Precision Engineering, Proceedings of the Eighteen Annual Meeting, p. 395-398 (2003)

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