

Novel Formulation of the 3D Large Deformation Beam Element for Dynamic Crash Analysis

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ABSTRACT

This paper presents a new Object Oriented Formulation of the FE algorithm that encompasses traditional FE, Super Element, experimental data and rigid body mechanics in a single calculation environment. The new formulation is implemented in a software for the dynamic crash simulation of an arbitrary 3D frame structure discretized into Super Elements and subjected to large dynamic crash loading. This paper presents basics of the general algorithm and element formulation. The theoretical portion of this paper is followed by a discussion of benchmark tests and results of the application to the real world structures.

1. Introduction

In recent years, it has become possible to use 3-dimensional (3D) CAD data to quickly create large-scale finite element models (FEM) that offer highly accurate reproductions of a physical component, due to the progress in capabilities of preprocessors and computers⁽¹⁾. Nonetheless, strength of materials-based beam models are still a quite effective method for preliminary calculations at the layout stage, when 3D CAD data have not yet been established.

Beam elements must fulfill the following requirements when used to analyze beam models of behavior during frame collapse in a collision:

- Consideration of elastic plastic issues
- Consideration of large deformation issues
- Consideration of dynamic issues
- Consideration of interactional collapse combining axial forces, bending, and torsion⁽²⁾
- Consideration of deformation of component cross section and high accuracy post-collapse (post-buckling)
- Simple data creation and short calculation times.

It is impossible to obtain all the considerations listed above in standard FEM formulations. The authors have proposed a new formulation which combines FEM, the Super Element method, and rigid body mechanics.

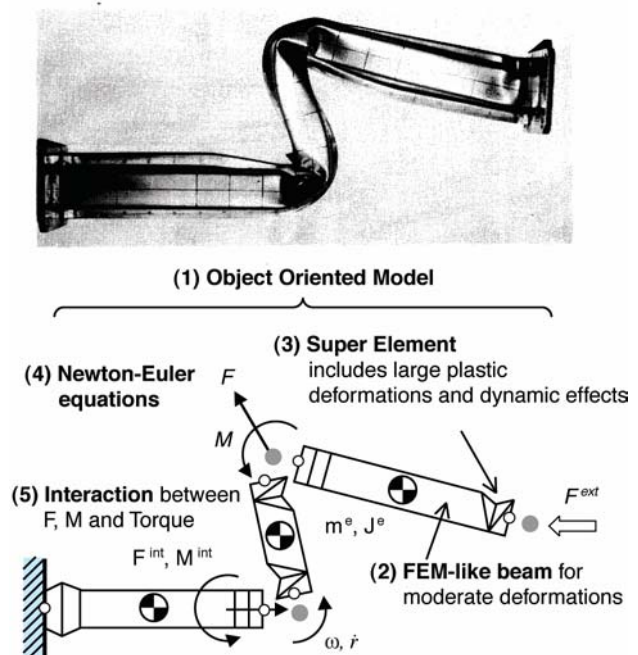


Fig. 1 Object oriented discretization of a thin-walled S-frame

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In addition, programming is done in an Object-Oriented Formulation (OOF)⁽³⁾ in order to ease the handling of non-linear issues. In this report, the formulation of what we call a 'superbeam' is explained, its utility and effectiveness are shown in comparisons of results with experiments and FEM.

2. Overview of Formulation

Figure 1 shows an overview of this formulation with an example of the bending of a thin walled curved beam.

- (1) Three numerical beam element models were created using OOF. A time integration scheme was employed with a central difference method.
- (2) Beam behavior in the elastic region was calculated using the Euler-Bernoulli beam theory.
- (3) Results were included with the Super Element method to provide results on beam elastic plastic issues, large deformation issues and dependence of yield stress on strain rate.
- (4) The Newton-Euler equations of motion were employed for considerations of dynamic issues. However, new nodes were established in different locations from the initial model nodes for use in calculating dynamic behavior and posture angle.
- (5) Interactional collapse combining axial force, bending, and torsion was modeled by constructing an element stiffness matrix based on the co-rotational method in continuous-body dynamics. Serial updating of the collapse surface was included in order to obtain high accuracy in the post-collapse region.

3. Solution of Equations of Motion

The principal of virtual velocity⁽⁴⁾⁻⁽⁷⁾ (or the principal of virtual work) shown in Eq. (1) was used to handle non-linear issues in this formulation.

$$\delta W(\delta u, u) \equiv \delta W^{in} - \delta W^{ext} + \delta W^{kin} = 0 \quad (1)$$

Here, δW^{in} is virtual internal work, δW^{ext} is virtual external work, and δW^{kin} is virtual kinetic energy. The equation of motion in Eq. (2) was obtained by discretizing Eq. (1).

$$M \ddot{u} = f^{ext} - f^{in} \quad (2)$$

Here, M is the mass matrix, $\ddot{u}$ is the nodal acceleration vector, and f^{in} and f^{ext} are the nodal internal force and nodal external forces, respectively.

Ordinarily, when Eq. (2) is solved with the central difference method, each element is treated in a step routine at each time step (element-by-element method). In this formulation, a method is proposed where each node is treated, and each element is treated, with complete separation between the two processes (Node-by-Node/Element-by-Element method).

Figure 2 shows how this method is related to the equation of motion. The inertia term on the left side of Eq. (2) is treated on a node-by-node basis according to the Newton-Euler equations of motion; the load terms on the right side are treated on an Element-by-Element basis by the Super Element method.

This method allows the two approaches of rigid body

mechanics and the Super Element method to be combined.

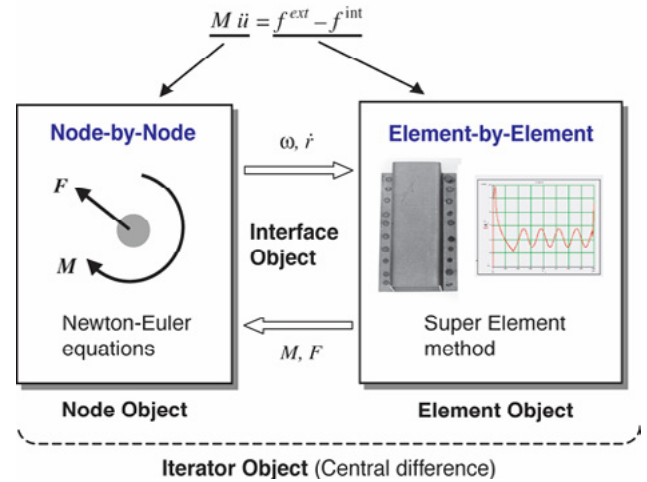


Fig. 2 Relationship between the equation of motion and Node-by-Node/Element-by-Element technique

4. Description of Objects Comprising OOP

Object-Oriented Programming (OOP)⁽⁸⁾ is an extremely effective way for programming non-linear programs. The present program is comprised of the four objects shown in Fig. 2. This chapter describes the operations in each of the objects.

4.1. Node Objects

When both forces and moments work on a node, the Newton-Euler equations of motion in Eq. (3) and (4) can be obtained^{(9), (10)}.

$$m \ddot{r} = F \quad (3)$$

$$J \dot{\omega} = M - \omega J \omega \quad (4)$$

Here, F and M are the forces and moments exerted on the node, m is the nodal mass, ω is the angular velocity, J is the inertial tensor, and $\ddot{r}$ is the nodal acceleration in the global coordinate system. The rotational motion in Eq. (4) is expressed in the local coordinate system.

For updating of new nodal locations, angular velocity ω , transformed to global coordinate, is found, and the instantaneous increase in angular position $\Delta\theta$ is calculated in Eq. (5).

$$\Delta\theta = \omega \Delta t \quad (5)$$

If this increase in angular position is used, the new location of the node can be expressed as a unit vector, as shown in Eq. (6).

$$\begin{aligned} \vec{i}_n &= \vec{i}_{n-1} + (a \times \vec{i}_{n-1}) \sin(\Delta\theta_n) + 2[a \times (a \times \vec{i}_{n-1})][1 - \cos(\Delta\theta_n)] \\ \vec{j}_n &= \vec{j}_{n-1} + (a \times \vec{j}_{n-1}) \sin(\Delta\theta_n) + 2[a \times (a \times \vec{j}_{n-1})][1 - \cos(\Delta\theta_n)] \\ \vec{k}_n &= \vec{i}_n \times \vec{j}_n \end{aligned} \quad (6)$$

4.2. Element Objects

When an object has deformed, the complete potential energy of the object J is defined as shown in Eq. (7) by the principal of the stationary potential energy^{(11), (12)}.

$$\frac{\partial J}{\partial \dot{q}_i} \delta \dot{q}_i \equiv 0 \quad (i = 1 \dots N) \quad (7)$$

Here, $\dot{q}$ is the generalized velocity and N is the total degrees of freedom. The complete potential energy J of the beam element is expressed in Eq. (8) using the principal of virtual work.

$$J(q, \dot{q}, \dot{h}) = Q\dot{q} - P(L_t, \dot{L}_t, \dot{h})\Delta L(q, \dot{q}) - M_\alpha(\phi_i, \dot{\phi}_i, \dot{h})\phi_\alpha(q, \dot{q}) \quad (8)$$

Here, Q is the external force (generalized force), P and M_α are the axial force and bending moment acting on the cross-section of the component, q is the generalized displacement, L_t is the component length after deformation, ϕ_k is the rotation, and $\dot{h}$ is a parameter expressing the condition of the axial force P and bending moment M_α .

In this formulation, the axial stiffness/bending stiffness matrix corresponding to the axial force and bending moment shown in Eq. (8) were derived using the co-rotational method used in continuous body dynamics. The ultimate element stiffness matrix was calculated using the principle of superposition.

4.2.1. Axial stiffness matrix

Figure 3 shows the post-deformation component length L_t defined in Eq. (9) using the local coordinate system.

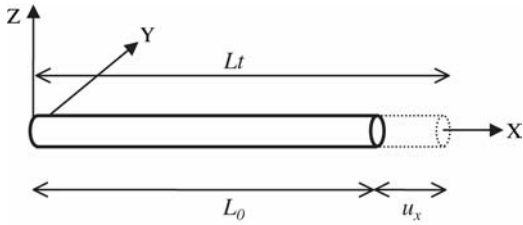


Fig. 3 Three-dimensional rod fixed at one end and subjected to large rotations and deformations

$$L(t) \equiv L_t = \sqrt{(L_0 + u_x)^2 + u_y^2 + u_z^2} \quad (9)$$

Here, L_0 is the initial component length, and u_x , u_y and u_z are the deformations of the component in the local coordinate system. The principle of virtual work shown in Eq. (10) is applied to these deformations u_x , u_y and u_z .

$$P \frac{\partial \Delta L}{\partial u_x} \delta u_x = 0, P \frac{\partial \Delta L}{\partial u_y} \delta u_y = 0, P \frac{\partial \Delta L}{\partial u_z} \delta u_z = 0 \quad (10)$$

The axial stiffness matrix shown in Eq. (11) can be derived using the results of Eq. (10)⁽¹³⁾. $P(t)$ in Eq. (11) is an axial collapse characteristic of the cross-section calculated with the Super Element method. While the beam is in the elastic region, an element stiffness matrix is constructed using the Euler-Bernoulli beam theory, but once the beam enters the plastic region, this is replaced with Eq. (11).

$$K_a(t) = \frac{P(t)}{L(t)} \begin{bmatrix} \frac{(L + u_x)}{L(t)} & 0 & 0 \\ u_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (11)$$

4.2.2. Bending stiffness matrix

According to the co-rotational method, the local coordinate system is defined at each point within an object and rotates with rotation of the object. Using a rotating local coordinate system eliminates the need to consider terms concerned with solid body rotation, such as the angular velocity tensor; only the deformation rate tensor must be included. The deformation speed tensor $\hat{D}_y$ is defined in Eq. (12)⁽⁴⁾.

$$\hat{D}_{ij} = \frac{1}{2} \left[\frac{\partial \hat{v}_i}{\partial \hat{x}_j} + \frac{\partial \hat{v}_j}{\partial \hat{x}_i} \right] \equiv \text{sym} \left[\frac{\partial \hat{v}_i}{\partial \hat{x}_j} \right] \equiv v_{i,j} \quad (12)$$

Here, $v_{i,j}$ the velocity in the local coordinate system. The bending stiffness matrix shown in Eq. (13) was derived from Eq. (12). Only 1/4 of the actual matrix is shown for simplification.

$$K_b(t) = \begin{bmatrix} \kappa u_y^2 & \kappa L_y u_y & 0 & 0 & 0 \\ 0 & -\kappa L_{xz}^2 & 0 & 0 & 0 \\ 0 & 0 & \kappa u_y^2 & 0 & 0 \\ 0 & 0 & 0 & \frac{M(\phi_y, \alpha_y) \cos(\alpha_y)}{\phi_z} & 0 \\ 0 & 0 & 0 & 0 & \frac{-M(\phi_y, \alpha_y) \sin(\alpha_y)}{\alpha_y} \end{bmatrix} \quad (13)$$

Here, $M(\phi, \alpha)$ is the bending collapse characteristic obtained by the Super Element method, ϕ and α are the rotational displacements, L_{xz} is the deformed length of the beam, and κ is the quantity defined in Eq. (14). The displacement vector $q(t)$ corresponding to Eq. (13) is shown in Eq. (15). The ultimate element stiffness matrix is obtained by combining Eq. (11) and Eq. (13) by the principle of superposition. That matrix allows expression of interactional collapse involving axial force and bending.

$$\kappa(t) = \frac{-M}{\sin(\theta) L_t^3 L_{xz}} \quad (14)$$

$$q(1)^T = \{u_x \ u_y \ u_z \ \phi_z \ \alpha_y\} \quad (15)$$

4.3. Interface Objects

Here, the interface object using a model of the combination of the two beam elements shown in Fig. 4 is explained. In the standard FEM, the characteristics of beam elements A and B are discretized at their common node I, and ultimately, the stiffness, displacement, velocity, internal forces, etc. are expressed at node I. If, in addition, it is attempted to enter equations for rigid body kinetics at I, it is very difficult to set up these equations. Therefore, in this formulation, as shown in Fig. 5, a new node J was established in addition to node I. Rigid body mechanics were expressed using this node J. Also, posture angle was calculated using this node J.

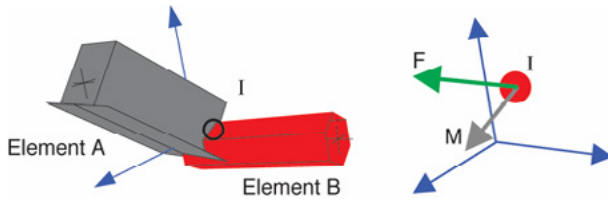


Fig. 4 Illustration of the basic paradigm of the FEM

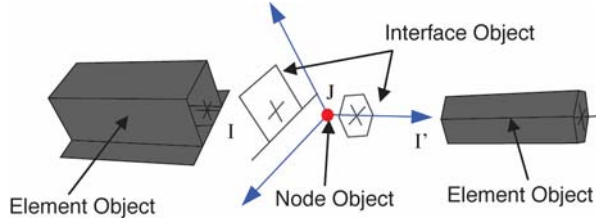


Fig. 5 Concept of the interface objects

Since the loads F and moments M were set constant at each time step when using a central difference scheme, as a result, it was possible to handle node J as a rigid body with no coupling with other locations.

This set of operations was carried out with node objects. Meanwhile, since element deformation was expressed at node I , calculations were handled to preserve equilibrium between the two nodes I and J using interface objects.

4.4. Iterator Object

An iterator object is a routine for carrying out time integration based on a central difference scheme. Figure 6 presents the overall flow.

- (1) The initial values are set for the mass and moment of inertia at the center of gravity of each element.
- (2) The initial values for velocity and angular velocity of each node are set and the mass and moment of inertia are calculated.
- (3) Treatment of each element is carried out on an element-by-element basis. From the velocity and angular velocity of each node, the resulting nodes and moments F^n and M^n are calculated and handed off to the next node-by-node step.
- (4) Treatment of each node is carried out on a node-by-node basis. Using the F^n and M^n values provided by step (3), the translational accelerations and angular velocities of each node are calculated according to the Newton-Euler equations and provided back to step (3).

5. Overview of Super Element Method

Beam plastic issues, large deformation issues and dependence of yield stress on strain rate are calculated using the Super Element method^{(14) - (18)} and combined into the stiffness matrices shown in Eq. (11) and (13).

Formulation of the Super Element method is basically comprised of three steps. The first step consists of assuming a plastic folding wavelength $2H$, a hinge line rolling radius r , and the deformation mode of wall faces and hinge lines indicated by 1-5, using data from the thin walled frame

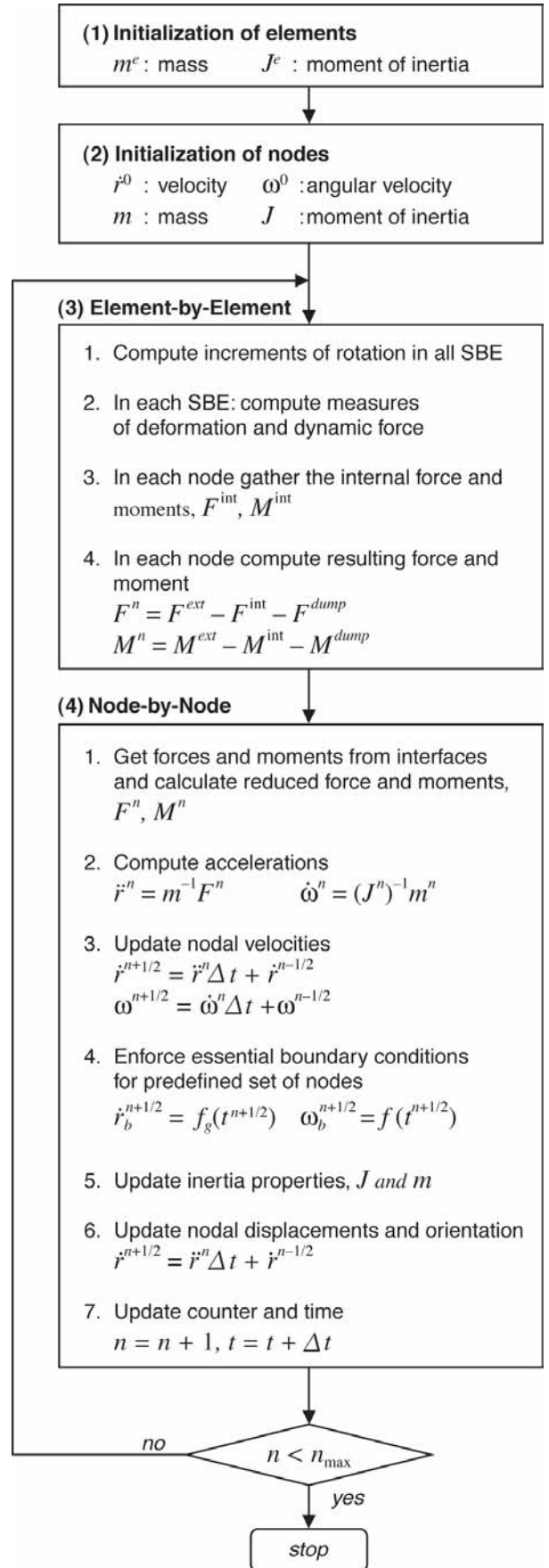
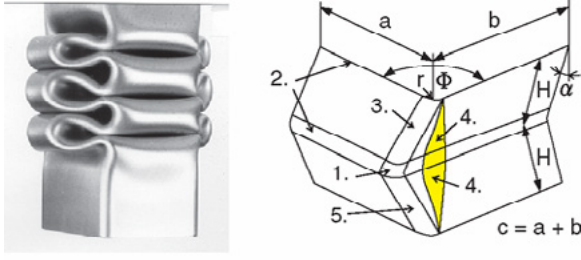


Fig. 6 Flow diagram of numerical procedure



1. Deformation of a 'floating' toroidal surface
2. Bending along stationary hinge lines
3. Rolling deformation
4. Opening of conical surface
5. Bending deformation

Fig. 7 Idealized deformation mechanisms

crushing test shown in Fig. 7. In the next step, the expression for the internal energy is assumed, as shown in Eq. (16), on the basis of plastic shell theory.

$$\dot{E}_{int} = \int_s N_o \dot{\epsilon}_1 dS + \sum_{i=1}^n \int_{L^i} M_o^i [\dot{\theta}_i] dl^i \quad (16)$$

Here, L^i is the length of the i^{th} hinge line, n is the total number of plastic hinge lines, N_o is the axial stress, and M_o^i is the bending stress. The first and second terms in Eq. (16) are calculated using Eq. (17) and (18).

$$E_{int}^N = \sigma_0^N(\bar{\epsilon}) \int_0^\alpha d\alpha \int_s \dot{\epsilon} dS \quad (17)$$

$$E_{int}^M = \sum_{i=1}^n \sigma_0^M(\bar{\epsilon}) \frac{t}{4} \int_0^\alpha d\alpha \int_{L^i} [\dot{\theta}_i] dl^i \quad (18)$$

Here, σ_0^N and σ_0^M are the flow stress⁽¹⁹⁾, α is a variable indicating the deformation process, and t is the sheet thickness. The mean crushing force P_m is defined as shown in Eq. (19), which is calculated using Eq. (16), (17) and (18).

$$P_m = \frac{t^2}{4} \left\{ \sigma_0^N(\bar{\epsilon}_1) A_1 \frac{r}{t} + \sigma_0^M(\bar{\epsilon}_2) A_2 \frac{C}{H} + \sigma_0^M(\bar{\epsilon}_3) A_3 \frac{H}{r} + \sigma_0^M(\bar{\epsilon}_4) A_4 \frac{H}{t} + \sigma_0^N(\bar{\epsilon}_5) A_5 \right\} \frac{2H}{\delta_{eff}} \quad (19)$$

Here, δ_{eff} is the effective crushing distance and A is a cross-sectional shape coefficient. In the last step, the mean crushing force P_m in the frame is obtained from Eq. (20), which is based on the principle of stationary potential energy.

$$P_m(H) = \frac{2H}{\delta_{eff}} \left[2\pi M_0 \frac{H}{t} + \frac{4\pi c M_0}{H} \right] \quad (20)$$

Here, M_0 is the fully plastic bending moment. The axial collapse characteristics and bending collapse characteristics can be calculated from the mean crushing force P_m in Fig. 8.

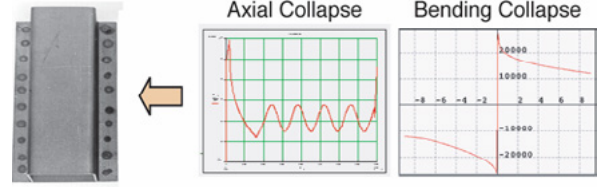


Fig. 8 Schematic illustration of the Super Element characteristics

5.1. Assessment of Collapse

Collapse assessment of beam elements in the pre-collapse region is carried out using Eq. (21).

$$\left(\frac{P}{P_{max}} \right)^a + \left(\frac{My}{My_{max}} \right)^a + \left(\frac{Mz}{Mz_{max}} \right)^a + \left(\frac{T}{T_{max}} \right)^a = 1 \quad (21)$$

Here, P_{max} , My_{max} , Mz_{max} and T_{max} are the maximum values for, respectively, the axial collapse force, bending collapse moments about the Y and Z axes, and torsional collapse moment in the beam. P , My , Mz and T are the loads occurring in the beam. Coefficient a is a variable determined by the thickness width ratio of the sheet. It is assumed that collapse occurs in the beam element when the left side of Eq. (21) reaches a value of 1.

Since P_{max} , My_{max} , Mz_{max} and T_{max} decrease in the post-collapse region due to deformation of the beam, Eq. (21) cannot be used here. Therefore, Eq. (22) was assumed for collapse assessment. Figure 9 shows the relationship between the collapse surface and the collapse force.

$$\left| \frac{P}{P_{max}(\phi)} \right| + \left| \frac{My}{My_{max}(\phi)} \right| + \left| \frac{Mz}{Mz_{max}(\phi)} \right| + \left| \frac{T}{T_{max}(\phi)} \right| = 1 \quad (22)$$

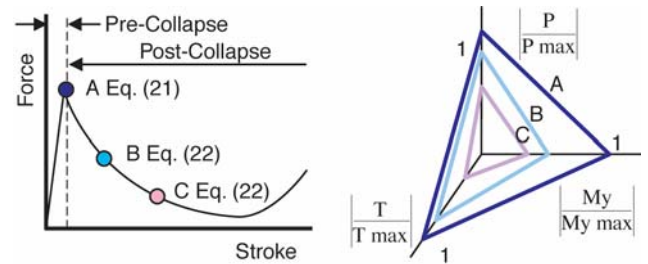


Fig. 9 Relationship between the interaction surface and the crushing characteristic

5.2. Analytical System

Figure 10 shows the analytical system for this beam model.

- (1) The axial collapse and bending collapse characteristics of the frame cross section are calculated using the cross section design software Crash Cad⁽¹⁸⁾ and the data are stored.
- (2) A 3D CAD model is created in the preprocessor. The element characteristics are set to the data saved in Crash Cad.
- (3) Once the boundary conditions have been set, the solver is set in operation.
- (4) The beam collapse behavior is investigated using the postprocessor.

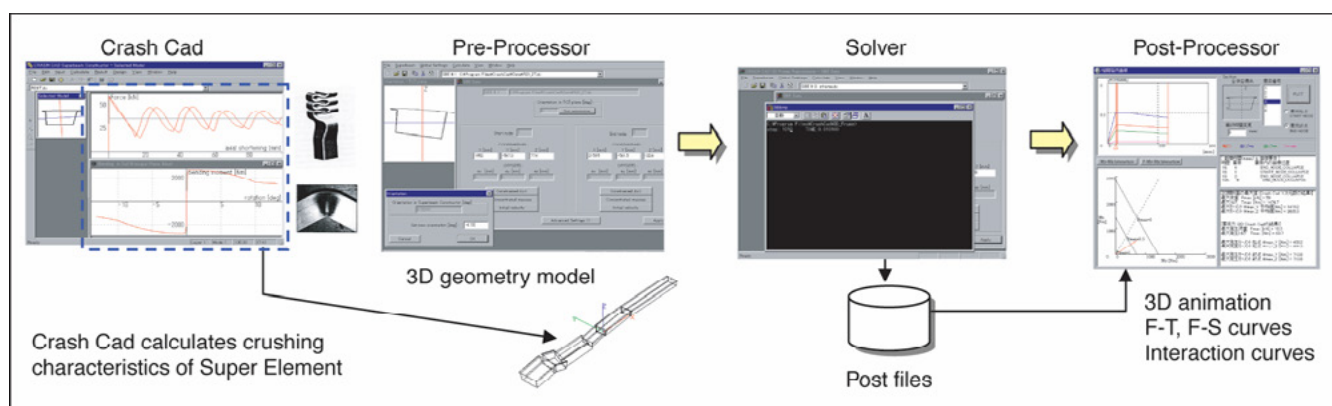


Fig. 10 Organization of the design/calculation environment for 3D large deformation beam systems

6. Examples of Use

6.1. Quasi-static Issue in an Aluminum Extrusion⁽²⁰⁾

Bending collapse under quasi-static load was analytically examined in an extruded aluminum specimen with both ends connected with a pin joint, as presented in Fig. 11. Two cross sections were used, designated A and B. The sheet in both was 2 mm thick. The material was series 6,000-T5, with a proof stress of 186 MPa, $E = 69$ MPa, and $\nu = 0.33$.

Force-stroke diagrams of the test and analytical results are shown in Fig. 12. It is clear that there is a high correlation between the test results and the results in this beam element for both the maximum force and the post-collapse region.

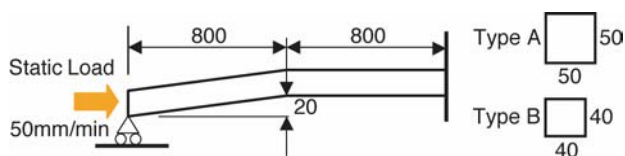


Fig. 11 Dimensions of the crooked extruded aluminum beam

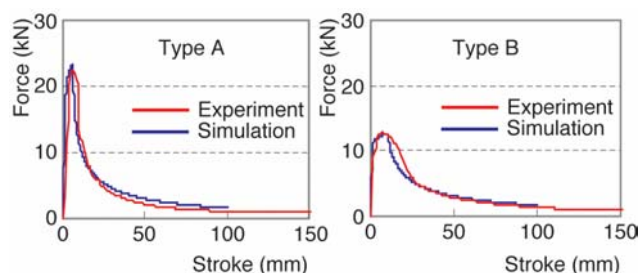


Fig. 12 Correlation of experimental and simulation results for the crooked aluminium beam shown in Fig. 11

6.2. Quasi-static Issue in Thin Walled Curved Beam^{(21),(22)}

This was an analysis of a thin-walled curved specimen with both ends completely fixed under a quasi-static load. The component shape and cross section are shown in Fig. 13. A hat section and flat plate were spot-welded to a flange of

the component and end plates were full-perimeter arc-welded onto each end of the component for attachment of the component to the test apparatus. The bend angles θ were 15° and 30° , wall thickness was 1.2 mm and the material was SPC.

Figure 14 shows a deformation mode test of a 15° bend specimen with a 100 mm stroke. It is clear that this beam element showed a high correlation with the specimen in results for collapse location and deformation angle.

Figure 15 shows force-stroke diagrams for the 15° and 30° bend specimens. Good accuracy was also found in this issue for maximum force and behavior in the post-collapse region.

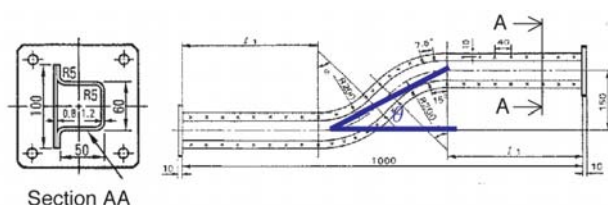


Fig. 13 Dimensions of the thin-walled steel S-frame

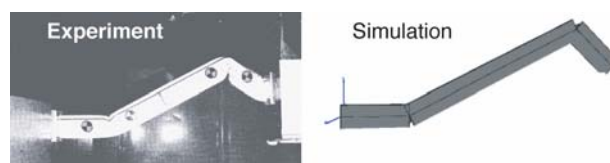


Fig. 14 Comparison of deformed modes at 100 mm stroke

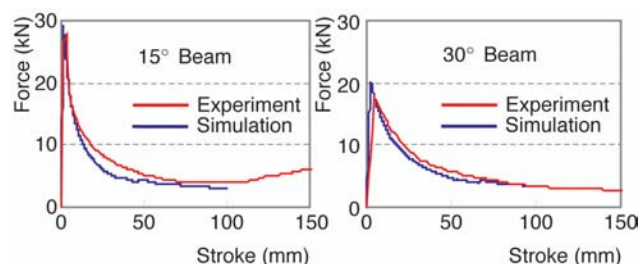


Fig. 15 Correlation of experimental and simulation results for the thin-walled S-frame shown in Fig. 13

6.3. Quasi-static Issue in Rear Frame

A quasi-static issue was analyzed using the rear frame from an actual automobile structure. The results of the beam element were compared with those from a FE model. Figure 16 illustrates the FE model and the superbeam model. The FE model was made completely from shell elements. The superbeam model was made from six elements.

Figure 17 shows the results for the predicted deformation modes in the FE and superbeam models after a stroke of 50 mm. It is clear that both models show good agreement in the deformation location and deformation mode. The maximum force during collapse was predicted at 19.5 kN in the FE model and 18.4 kN in the superbeam model, quite close predictions.

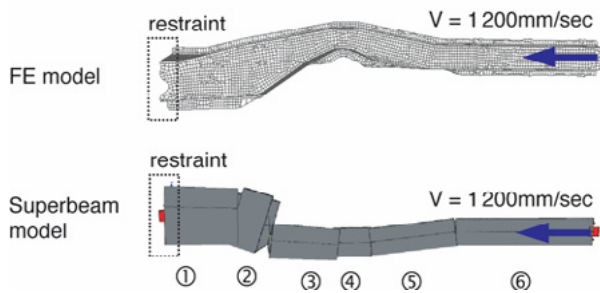


Fig. 16 Boundary and loading conditions for FEM and Superbeam models

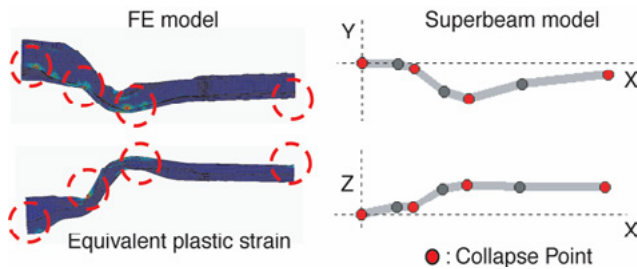


Fig. 17 Deformed configurations of FEM and Superbeam

6.4. Dynamic Torsion Issue in Box Frame

An analysis was performed with a 90° frame corner made of two square cross section beams, subjected to a dynamic load (Fig. 18). The purpose of this analysis was to test the accuracy of this formulation in a collapse issue where torsional deformation is a dominant factor.

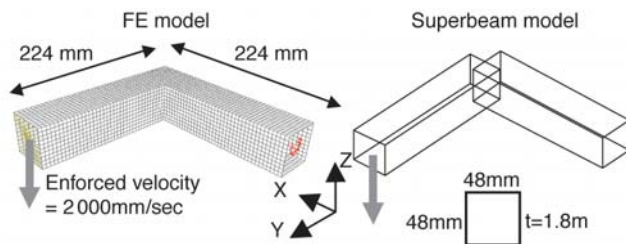


Fig. 18 Boundary and loading conditions for the FEM and the Superbeam model of a thin-walled torque box

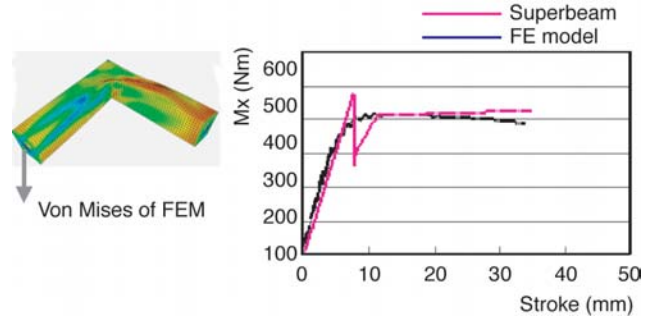


Fig. 19 Correlation of FEM and Superbeam results for the thin-walled torque box shown in Fig. 18

Figure 19 shows the force-stroke diagram. A fairly good correlation was found with the FEM. The superbeam model showed a vibration at a stroke of 8 mm, but this was caused by the modeling of the structure with two beam elements. This issue was solved when the model was divided more finely.

6.5. Dynamic Issue in Thin Walled Curved Beam⁽²³⁾

A dynamic issue was analyzed using the same model as in 6.2. Figure 20 shows a schematic view of the dynamic test apparatus. Components with a 45° bend were fixed to the front of a dolly and crashed into a barrier at 30 kph.

Figure 21 shows the force-stroke diagram. The measurement of the initial maximum force in the test was affected by the weight of the plate attached to the tip of the test specimen⁽²³⁾, by the sampling pitch of the load cell and perhaps by other factors. It differed from the result for maximum force given by the beam model. A good correlation between the results was found in the post-collapse region, however.

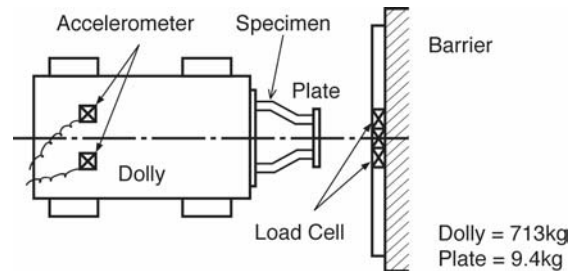


Fig. 20 Schematic diagram of dynamic experiment

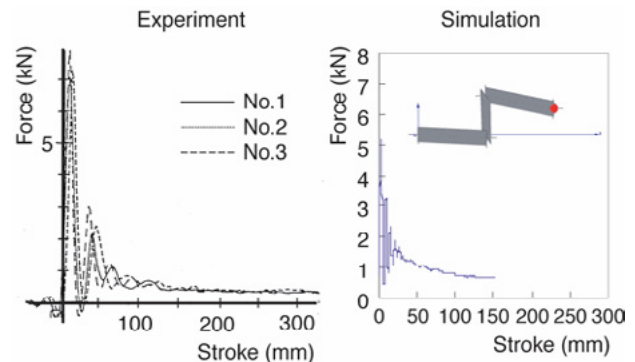


Fig. 21 Results of experiments and Superbeam model simulation for an S-frame shown in Fig. 20

6.6. Comparison of CPU Time

The calculation times for the rear frame test in 6.3 were compared. The FE model was comprised of about 5,000 shell elements. On a supercomputer, a calculation time of about 22 h was necessary for a deformation of up to 200 mm.

The superbeam model required 68 seconds for calculations for deformation of up to 200 mm on a PC.

7. Summary

For this report, a new formulation combining FEM, the Super Element method, and rigid-body mechanics was created to analyze dynamic collapse issues in a beam element. The results of this formulation were as follows:

- (1) It was confirmed that this 'superbeam' element accorded well with experimental models. At the same time, it was proved that the Super Element method is effective.
- (2) A tool was developed which enables preliminary investigation, at the layout stage, of component collapse characteristics using cross section data.
- (3) By using this formulation, it was judged possible to develop non-linear shell elements which provide calculation results in short times.

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