

The Effect of Geometry on Stiffness and Deformation in Exhaust Manifold for In-line 4-Cylinder Engines

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ABSTRACT

Investigations were conducted on a quasi quantitative analysis method for exhaust manifold stiffness and deformation that is useful when designing exhaust manifolds for in-line 4 cylinder engines. This method analyzes the exhaust manifold as a combination of beams, making it possible to obtain the effects of the dimensions and shapes of major parts of exhaust manifold on stiffness and deformation. These results clarified the effects of various exhaust manifold shape characteristics (aspect ratio, branch ratio, offset ratio) on non-dimensional stiffness, and also the effects of aspect ratio on the non-dimensional warping quantity. In addition, the effects of each of the aspect ratio, collective part different angle, branch ratio and offset ratio on non-dimensional stiffness and deformation when the dimensions and shape of each exhaust manifold part are changed could also be clarified by obtaining the degree of effects of each factor. The proposed method is effective for determining and investigating the validity of major approximate dimensions, particularly when designing new exhaust manifolds with limited outer dimensions. Exhaust manifolds can be efficiently developed by performing CAE calculations on exhaust manifold shapes that have been determined using this method.

1. Introduction

The demand for automobile engines with cleaner exhaust emissions and lower fuel consumption is increasing throughout the world. As a consequence, conspicuous efforts are being made, for example, to reduce the heat capacity of

engine exhaust systems (reducing wall thickness) in order to raise the temperature of the catalytic converter to the catalyst activation temperature as quickly as possible when starting the engine. Other efforts involve the use of lean burning to raise the exhaust gas temperature for the purpose of lowering fuel consumption, and so on. These approaches all impose

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extremely harsh conditions on the exhaust manifold. Many problems arising during engine development can therefore be traced to the exhaust system, and the requirement for even further reduction of the development time means that there is strong demand for optimized design of the exhaust manifold in the initial stages of development.

It has recently become common to use CAE to estimate the stress, deformation, and other factors affecting each part of the exhaust manifold. When using CAE to design a new exhaust manifold, however, it becomes necessary to hypothesize a temperature for each of its parts in order to make the calculations. Those actual calculations cannot be performed unless the geometry of the exhaust manifold is determined in advance.

When designing a new exhaust manifold, therefore, it is crucial to understand beforehand the effects that the dimensions and shapes of the exhaust manifold parts will have on stiffness and deformation. Methods are being sought that will, in other words, make it possible to clearly grasp the major approximate dimensions of the exhaust manifold parts and to investigate their shapes in terms of stiffness and deformation, and to do these things in a short period of time at the designer's desk. In the interests of development efficiency, it is thought that the best method of designing a new exhaust manifold is first to use a quasi quantitative method to derive the approximate size. This then provides a basis for more precise calculations using CAE.

The authors formerly reported on a method for evaluating the reliability of an exhaust manifold using the restraint ratio.^{(1), (2)} The present research deals with the stage prior to that by proposing a method for determining the effects of the dimensions of the various exhaust manifold parts on its stiffness and displacement ratio. In other words, this method can be used on the desktop to approximate the basic major dimensions and geometry when designing a new exhaust manifold for a 4-cylinder engine. Using a method of this kind to estimate the basic dimensions and geometry, for using those as a basis for CAE calculation of the restraint ratio and other factors, makes it possible to determine the final dimensions and geometry of an exhaust manifold within a short period of time.

The methods used previously for analysis relating to exhaust manifolds for in-line 2-cylinder engines served as the basis for the present analysis method.⁽³⁾ These methods were extended for application to exhaust manifolds for 4-cylinder engines. The present approach also involved analysis and investigation in even greater detail.

Nomenclature

a_A	Distance from the exhaust manifold center to the No. 2 port and No. 3 port entrances after deformation (see Fig. 3)
a_B	Distance from the exhaust manifold center to the No. 1 port and No. 4 port entrances after deformation (see Fig. 3)
b	Branch ratio ($= U/y$)
c	Offset ratio ($= L/y$)
d_{A1}	Deformation resulting from the moment generated in No. 2 port and No. 3 port
d_{A2}	Deformation resulting from the load applied in No. 2 port and No. 3 port

d_{B1}	Deformation resulting from the moment generated in No. 1 port and No. 4 port
d_{B2}	Deformation resulting from the load applied in No. 1 port and No. 4 port
dx_A	Amount of deformation in No. 2 port and No. 3 port (see Fig. 3)
dx_B	Amount of deformation in No. 1 port and No. 4 port (see Fig. 3)
E	Modulus of longitudinal elasticity
G_{XA}	Stiffness of collective part of No. 2 port and No. 3 port
G_{XB}	Stiffness of collective part of No. 1 port and No. 4 port
h_A	Length of No. 2 port and No. 3 port after deformation (see Fig. 3)
h_B	Length of No. 1 port and No. 4 port after deformation (see Fig. 3)
I	Geometrical moment of inertia (geometrical moment of attachment flange + geometrical moment of port branch + geometrical moment of collective part)
L	Distance from port entrance to port collective part (see Fig. 3)
M_0	Moment operating in each port ($= WL$)
m	Aspect ratio ($= L/U$)
r	Geometrical center of the opening of each port when the exhaust manifold is deformed (see Fig. 3)
$S_{A\theta}$	See Eq. (51)
S_{Am}	See Eq. (52)
$S_{B\theta}$	See Eq. (56)
S_{Bm}	See Eq. (57)
$T_{A\theta}$	See Eq. (37)
T_{Am}	See Eq. (38)
T_{Ab}	See Eq. (39)
T_{Ac}	See Eq. (40)
$T_{B\theta}$	See Eq. (44)
T_{Bm}	See Eq. (45)
T_{Bb}	See Eq. (46)
T_{Bc}	See Eq. (47)
U	Length of exhaust manifold (see Fig. 3)
W	Load acting upon each port
y	Distance from port entrance to the port collective part (see Fig. 3)
ΔZ_A	Quantity of warp in No. 2 port and No. 3 port (see Fig. 3)
$\Delta Z_A'$	Quantity of apparent warp in No. 2 port and No. 3 port
ΔZ_B	Quantity of warp in No. 1 port and No. 4 port (see Fig. 3)
$\Delta Z_B'$	Quantity of apparent warp in No. 1 port and No. 4 port
θ	Difference angle of collective part (see Fig. 3)
Ψ_A	Opening angle of No. 2 port and No. 3 port when deformation occurs (see Fig. 3)
Ψ_B	Opening angle of No. 1 port and No. 4 port when deformation occurs (see Fig. 3)

2. Analysis

The exhaust manifold is considered as a combination of beams to determine its stiffness. The method also seeks to identify the deformation caused by the moment generated in the various parts of the exhaust manifold, and the deformation caused by the load acted. These are combined to calculate the total deformation.

Fig. 1 shows an actual form and schematic model of the exhaust manifold for a 4-cylinder engine that is the object of this investigation. This exhaust manifold is considered to be left-right symmetrical. That is, the No. 1 port and No. 4 port parts are symmetrical with respect to the center line of the exhaust manifold, and the No. 2 port and No. 3 port parts likewise. Deformation is caused by WL and W .

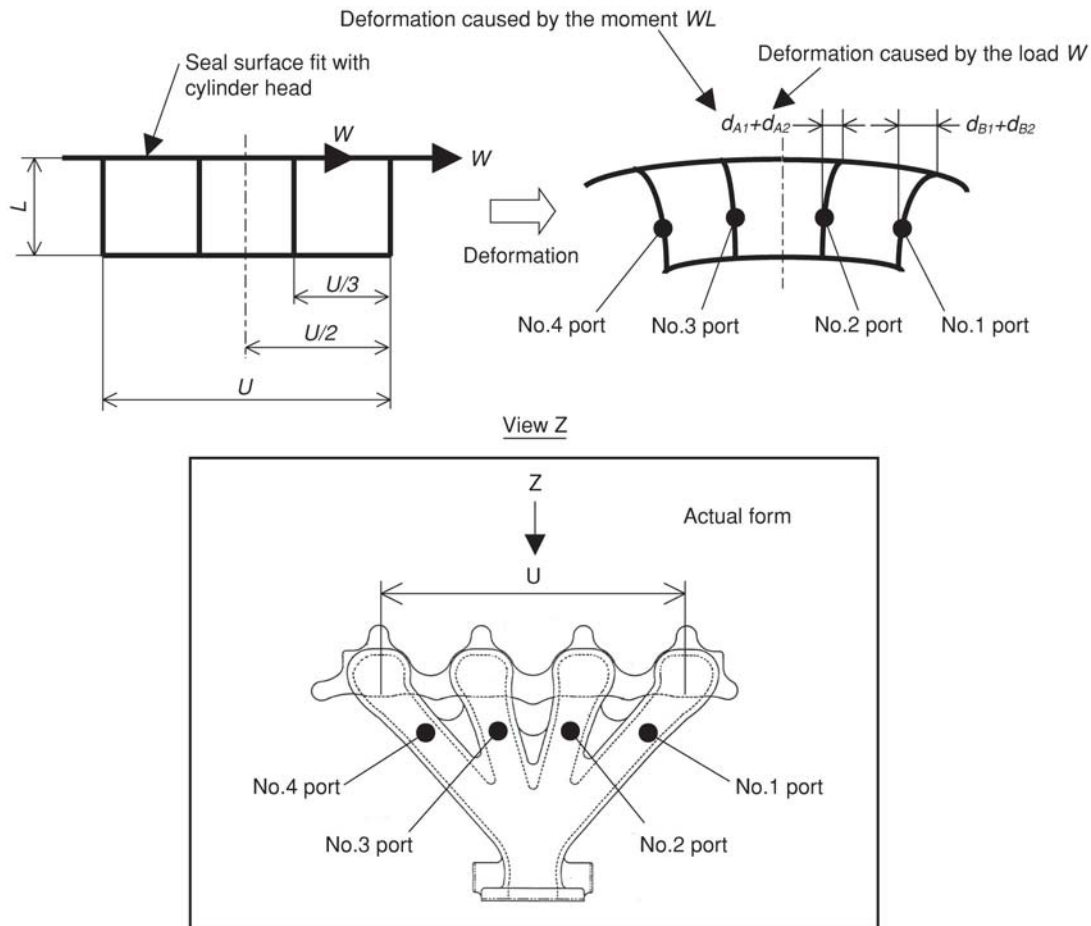


Fig. 1 Actual form and schematic model of exhaust manifold for in-line 4 cylinder engine. Deformation is caused by the moment WL and the load W .

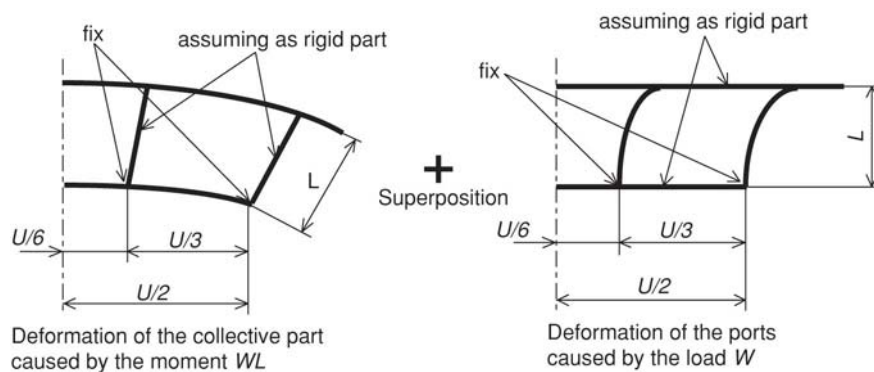


Fig. 2 Superposition of deformation of the collective part caused by the moment WL and deformation of the ports caused by the load W .

Fig. 2 is a schematic representation of the deformation caused by WL in the collective part and the deformation caused by W in the individual ports. The deformation of the collective part caused by the moment and the deformation of the individual ports caused by the load in this way are combined.

The section below introduces three non-dimensional numbers that represent geometrical properties of the exhaust manifold. These numbers are commonly applied in analysis and evaluation of the various dimensions and shapes of the exhaust manifold by a common index. They are the aspect ratio, branch ratio, and offset ratio.

The model used in this analysis is shown in Fig. 3.

2.1. Effect of Aspect Ratio and Collective Part Difference Angle on Stiffness

The case of the simplest geometry, where $\theta = 0^\circ$, will be considered first.

d_{A1} is the deformation that occurs when the moment WL operates on the end of a beam of length $U/6$. This can be expressed by Eq. (1):

$$d_{A1} = M_0(U/6)^2/2EI = (WL/2EI)(U/6)^2 \quad (1)$$

Here EI is the flexural rigidity.

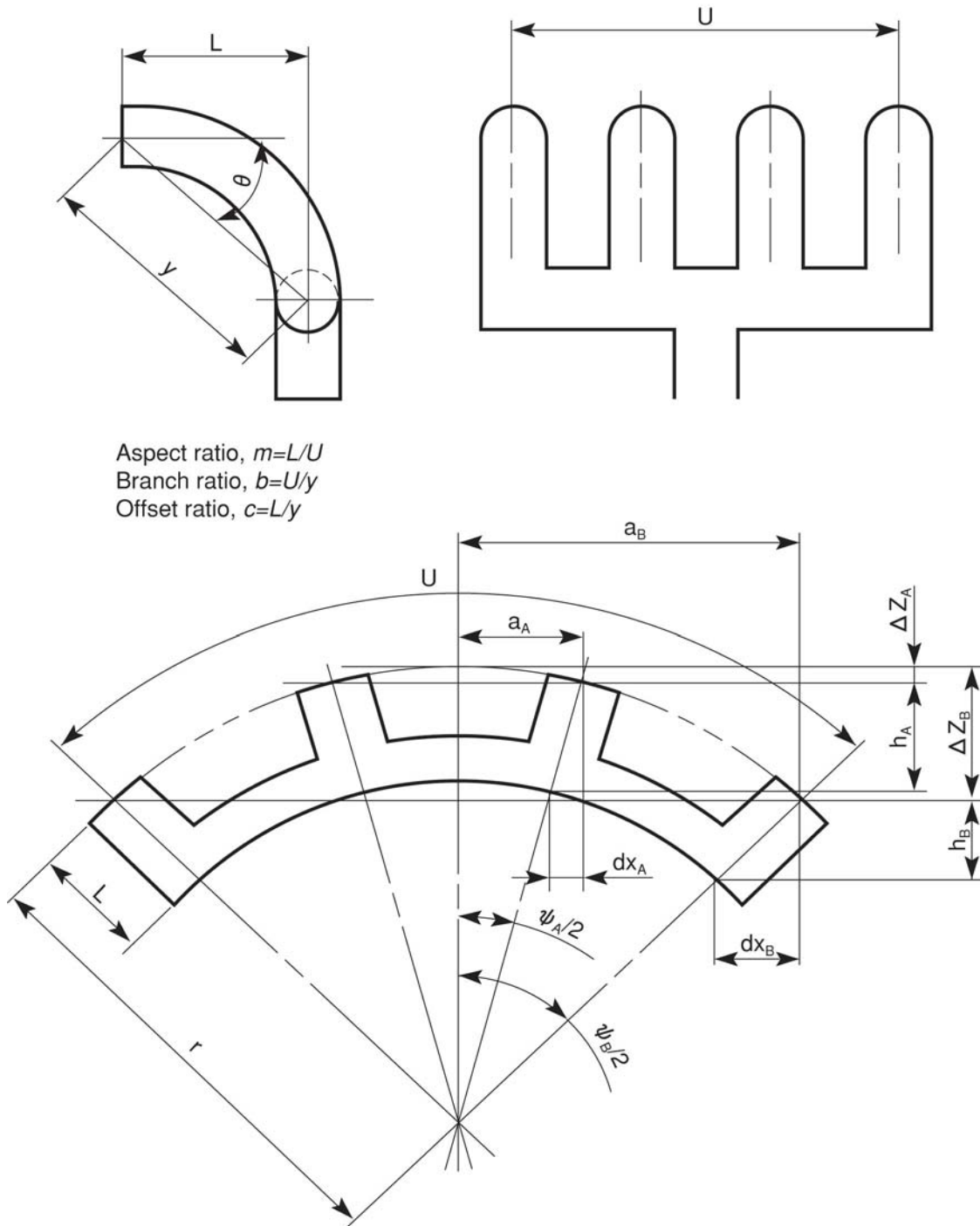


Fig. 3 Model of the exhaust manifold used for the analysis

Similarly, d_{B1} is the deformation that occurs when the moment WL operates on the end of a beam of length $U/2$. This can be expressed by Eq. (2):

$$\begin{aligned} d_{B1} &= M_0(U/2)^2/2EI \\ &= (WL/2EI)(U/2)^2 \end{aligned} \quad (2)$$

Consequently, the deformation $d_{A1}+d_{B1}$ of the ends of the collective part caused by the moments operating on each branch can be defined using Eqs. (1) and (2):

$$\begin{aligned} d_{A1}+d_{B1} &= (WL/2EI)(U/6)^2 \\ &\quad + (WL/2EI)(U/2)^2 \\ &= (5WLU^2)/(36EI) \end{aligned} \quad (3)$$

Next, d_{A2} and d_{B2} can represent the deformation that occurs when W operates on the end of beams of length L that have one end fixed, so that:

$$d_{A2} = d_{B2} = (WL^3)/(3EI) \quad (4)$$

Therefore, from Eqs. (1) and (4):

$$\begin{aligned} dx_A = d_{A1}+d_{A2} &= (WL)(U/6)^2/(2EI) \\ &\quad + (WL^3)/(3EI) \\ &= (WL)(U^2+24L^2)/(72EI) \end{aligned} \quad (5)$$

And from Eqs. (3) and (4):

$$\begin{aligned} dx_B = d_{A1}+d_{B1}+d_{B2} &= (5WLU^2)/(36EI) \\ &\quad + (WL^3)/(3EI) \\ &= (WL)(5U^2+12L^2)/(36EI) \end{aligned} \quad (6)$$

From Eqs. (5) and (6), the stiffness of the collective part will be:

$$\begin{aligned} Gx_A &= W/dx_A \\ &= (72EI)/[L(U^2+24L^2)] \end{aligned} \quad (7)$$

$$\begin{aligned} Gx_B &= W/dx_B \\ &= (36EI)/[L(5U^2+12L^2)] \end{aligned} \quad (8)$$

Since $m = L/U$, L can be eliminated from Eq. (7), leaving:

$$\begin{aligned} Gx_A &= (72EI)/[mU(U^2+24m^2U^2)] \\ &= (EI/U^3)\{72/[m(1+24m^2)]\} \end{aligned}$$

Therefore, the non-dimensional stiffness of No. 2 port and No. 3 port is:

$$Gx_A U^3/EI = 72/[m(1+24m^2)] \quad (9)$$

Similarly, L can be eliminated from Eq. (8), giving:

$$\begin{aligned} Gx_B &= (36EI)/[mU(5U^2+12m^2U^2)] \\ &= (EI/U^3)\{36/[m(5+12m^2)]\} \end{aligned}$$

Consequently, the non-dimensional stiffness of No. 1 port and No. 4 port is:

$$Gx_B U^3/EI = 36/[m(5+12m^2)] \quad (10)$$

Next the effect of θ will be considered.

Substituting $L/\cos\theta$ for L in Eqs. (1) through (8), these become:

$$d_{A1} = (WL/2EI)(U/6)^2 \cos^{-1}\theta \quad (11)$$

$$d_{B1} = (WL/2EI)(U/2)^2 \cos^{-1}\theta \quad (12)$$

$$\therefore d_{A1}+d_{B1} = (5WL/36EI)U^2 \cos^{-1}\theta \quad (13)$$

$$d_{A2} = d_{B2} = (WL/3EI) \cos^3\theta \quad (14)$$

$$\begin{aligned} \therefore dx_A &= d_{A1}+d_{A2} \\ &= (WL/72EI)[(U^2/\cos\theta) \\ &\quad + (24L^2/\cos^3\theta)] \end{aligned} \quad (15)$$

$$\begin{aligned} dx_B &= d_{A1}+d_{B1}+d_{B2} \\ &= (WL/36EI)[(5U^2/\cos\theta) \\ &\quad + (12L^2/\cos^3\theta)] \end{aligned} \quad (16)$$

$$\begin{aligned} \therefore Gx_A &= W/dx_A \\ &= (72EI)/\{L[(U^2/\cos\theta) \\ &\quad + (24L^2/\cos^3\theta)]\} \end{aligned} \quad (17)$$

$$\begin{aligned} Gx_B &= W/dx_B \\ &= (36EI)/\{L[(5U^2/\cos\theta) \\ &\quad + (12L^2/\cos^3\theta)]\} \end{aligned} \quad (18)$$

Since $m = L/U$, L can be eliminated from Eqs. (17) and (18) and the non-dimensional stiffness derived by:

$$\begin{aligned} Gx_A U^3/EI &= 72/\{m[(1/\cos\theta) \\ &\quad + (24m^2/\cos^3\theta)]\} \end{aligned} \quad (19)$$

$$\begin{aligned} Gx_B U^3/EI &= 36/\{m[(5/\cos\theta) \\ &\quad + (12m^2/\cos^3\theta)]\} \end{aligned} \quad (20)$$

Eqs. (19) and (20) can be used with θ as a parameter to plot the non-dimensional stiffness $Gx_A U^3/EI$ and $Gx_B U^3/EI$ with respect to m . This yields Fig. 4 and Fig. 5, respectively. Plotting these again with respect to θ yields Fig. 6 and Fig. 7, respectively.

It can be seen in Fig. 4 through Fig. 7 that the stiffness increases as θ becomes smaller. As m becomes larger, however, the effect of θ becomes negligible. As m grows smaller, the stiffness increases exponentially. These observations can be made alike in No. 1 port and No. 4 port, and No. 2 port and No. 3 port. Comparison of the No. 1 and No. 4 ports with the No. 2 and No. 3 ports shows that, under identical conditions, the No. 2 and No. 3 ports have stiffness several times greater than the No. 1 and No. 4 ports (especially when m is small).

2.2. Changes in Non-dimensional Stiffness with Respect to the Branch Ratio

Substituting $m = \cos\theta/b$ in Eq. (9) yields:

$$Gx_A U^3/EI = 72b^3/(b^2 \cos\theta + 24 \cos^3\theta) \quad (21)$$

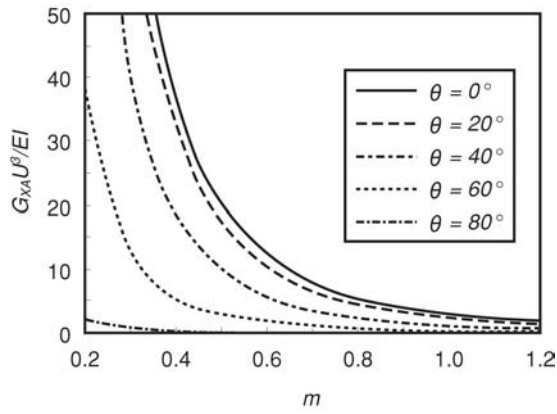


Fig. 4 Non-dimensional stiffness rigidity for No.2 and No.3 ports vs. aspect ratio “ m ”. Non-dimensional stiffness rigidity increases as “ m ” and θ become smaller.

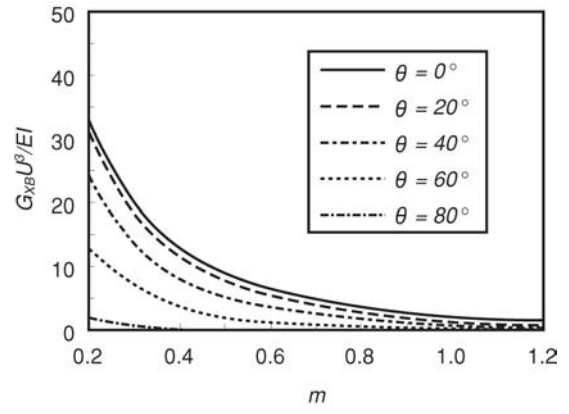


Fig. 5 Non-dimensional stiffness rigidity for No.1 and No.4 ports vs. aspect ratio “ m ”. Non-dimensional stiffness rigidity increases as “ m ” and θ become smaller. In the case of a smaller “ m ”, stiffness rigidity for the No.1 and No.4 ports is quite small compared with that of the No.2 and No.3 ports (Fig. 4).

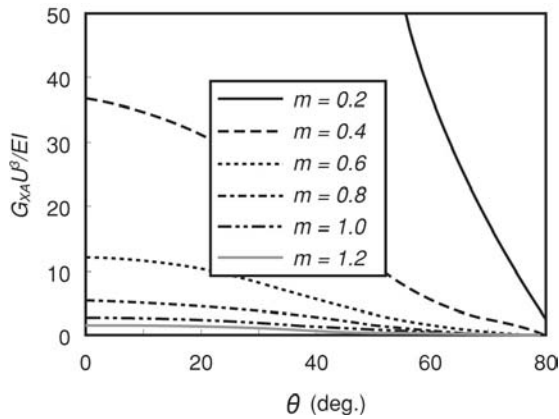


Fig. 6 Non-dimensional stiffness rigidity for No.2 and No.3 ports vs. deviation angle θ .

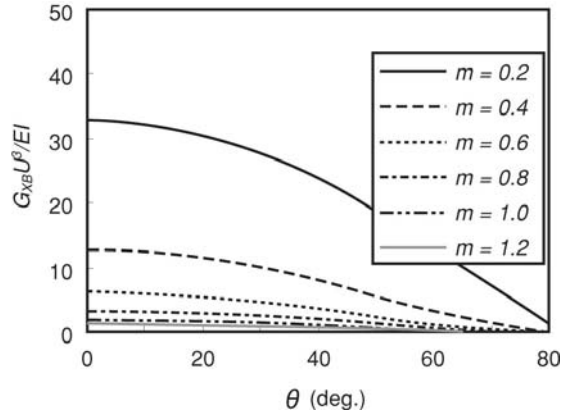


Fig. 7 Non-dimensional stiffness rigidity for No.1 and No.4 ports vs. deviation angle θ .

Similarly, in Eq. (10):

$$G_{xB}U^3/EI = 36b^3/(5b^2\cos\theta + 12\cos^3\theta) \quad (22)$$

Eqs. (21) and (22) can be used with θ as a parameter to plot the non-dimensional stiffness $G_{xA}U^3/EI$ and $G_{xB}U^3/EI$ with respect to b . This yields Fig. 8 and Fig. 9, respectively. Plotting these again with respect to θ yields Fig. 10 and Fig. 11, respectively.

It can be seen in Fig. 8 through Fig. 11 that the stiffness increases when θ becomes larger, if b remains the same. As b becomes smaller, however, that difference diminishes. As b grows larger, the stiffness increases exponentially. These observations can be made alike in No. 1 port and No. 4 port, and No. 2 port and No. 3 port. Comparison of the stiffness of the No. 1 and No. 4 ports with the No. 2 and No. 3 ports shows that, under identical conditions, the No. 2 and No. 3 ports have stiffness several times greater than the No. 1 and No. 4 ports (especially when b is small).

2.3. Changes in Non-dimensional Stiffness with Respect to the Offset Ratio

Eqs. (17) and (18) can be used with m as a parameter to plot the non-dimensional stiffness with respect to the offset ratio. The results are shown in Fig. 12 and Fig. 13, respectively.

The stiffness increases when m becomes smaller, if the offset ratio remains the same. When the offset ratio is smaller, however, that difference diminishes. As the offset ratio grows larger, the stiffness increases exponentially. These observations can be made alike in No. 1 port and No. 4 port, and No. 2 port and No. 3 port. Comparison of the stiffness of the No. 1 and No. 4 ports with the No. 2 and No. 3 ports shows that, under identical conditions, the No. 2 and No. 3 ports have stiffness several times greater than the No. 1 and No. 4 ports (especially when the offset ratio is small).

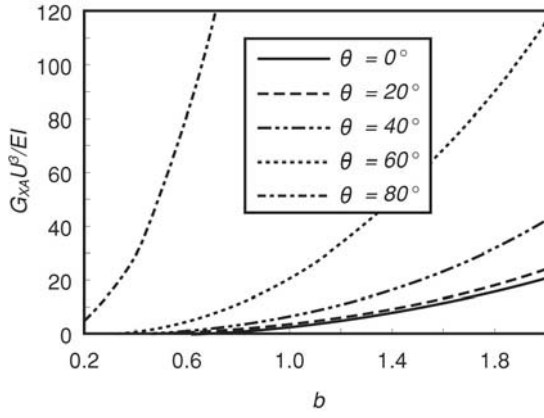


Fig. 8 Non-dimensional stiffness rigidity for No.2 and No.3 ports vs. branch ratio “ b ”. Non-dimensional stiffness rigidity increases as “ b ” and θ become larger.

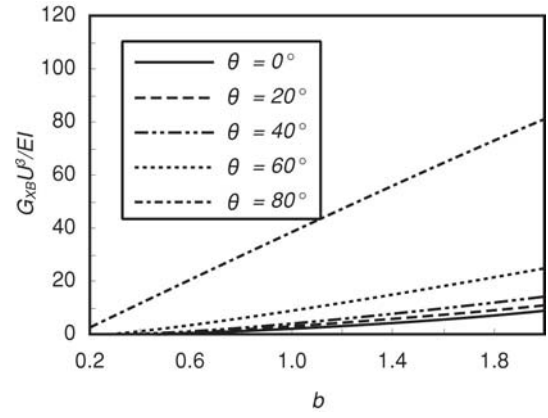


Fig. 9 Non-dimensional stiffness rigidity for No.1 and No.4 ports vs. branch ratio “ b ”. Non-dimensional stiffness rigidity increases as “ b ” and θ become larger. Especially in the case of large “ b ”, stiffness rigidity for the No.1 and No.4 port is smaller than that for the No.2 and No.3 ports.

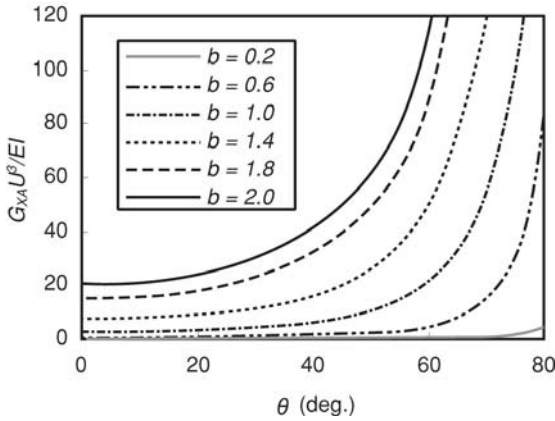


Fig. 10 Non-dimensional stiffness rigidity for No.2 and No.3 ports vs. deviation angle θ .

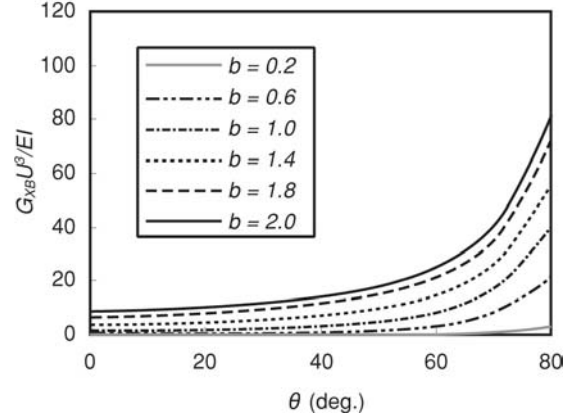


Fig. 11 Non-dimensional stiffness rigidity for No.1 and No.4 ports vs. deviation angle θ .

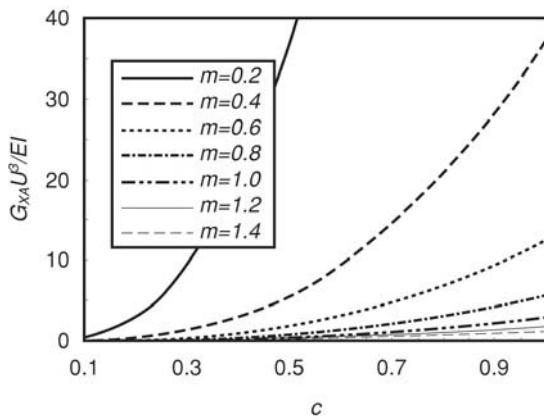


Fig. 12 Non-dimensional stiffness rigidity for No.2 and No.3 ports vs. offset ratio “ c ”. Non-dimensional stiffness rigidity increases as “ c ” becomes larger and “ m ” becomes smaller.

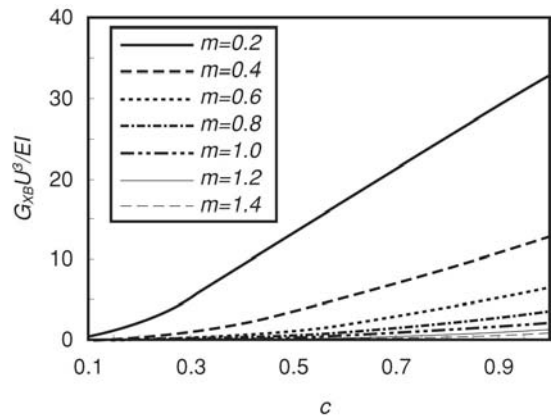


Fig. 13 Non-dimensional stiffness rigidity for No.1 and No.4 ports vs. offset ratio “ c ”. Non-dimensional stiffness rigidity increases as “ c ” becomes larger and “ m ” becomes smaller. Non-dimensional stiffness rigidity for the No.1 and No.4 ports is smaller than that of the No.2 and No.3 ports.

2.4. Quantity of Warp

i) ΔZ_A

From Fig. 2 and Fig. 3:

$$\begin{aligned} r^2 &= a_A^2 + (r - \Delta Z_A)^2 \\ &= a_A^2 + r^2 - 2r\Delta Z_A + \Delta Z_A^2 \end{aligned}$$

Here, since $\Delta Z_A \ll U$, it follows that:

$$\begin{aligned} r^2 &= a_A^2 + r^2 - 2r\Delta Z_A \\ \therefore 2r\Delta Z_A &= a_A^2 \\ a_A &= [(U/6)^2 - \Delta Z_A^2]^{1/2} \approx U/6 \end{aligned}$$

Consequently:

$$\begin{aligned} 2r\Delta Z_A &= (U/6)^2 \\ \therefore r &= (U/6)^2 / (2\Delta Z_A) \end{aligned} \quad (23)$$

When $\Psi_A \ll 1$:

$$dx_A/h_A = \tan(\Psi_A/2) \approx \Psi_A/2$$

Therefore:

$$\Psi_A/2 = dx_A/h_A = dx_A/L \quad (24)$$

Here, the length $U/6$ of the arc with radius r and angle $\Psi_A/2$ will be (see Fig. 3):

$$U/6 = r(\Psi_A/2) \quad (25)$$

Substituting Eqs. (23) and (24) in Eq. (25) yields:

$$\begin{aligned} U/6 &= [(U/6)^2 / (2\Delta Z_A)] (dx_A/L) \\ &= (U^2 dx_A) / (72L\Delta Z_A) \\ \therefore \Delta Z_A &= (1/12)(U/L) dx_A \\ &= [1/(12m)] dx_A \end{aligned} \quad (26)$$

ii) ΔZ_B

Similarly:

$$\begin{aligned} r^2 &= a_B^2 + (r - \Delta Z_B)^2 \\ &= a_B^2 + r^2 - 2r\Delta Z_B + \Delta Z_B^2 \end{aligned}$$

Here, since $\Delta Z_B \ll U$, it follows that:

$$\begin{aligned} r^2 &= a_B^2 + r^2 - 2r\Delta Z_B \\ \therefore 2r\Delta Z_B &= a_B^2 \\ a_B &= [(U/2)^2 - \Delta Z_B^2]^{1/2} \approx U/2 \end{aligned}$$

Consequently:

$$\begin{aligned} 2r\Delta Z_B &= (U/2)^2 \\ \therefore r &= (U/2)^2 / (2\Delta Z_B) \end{aligned} \quad (27)$$

When $\Psi_B \ll 1$:

$$dx_B/h_B = \tan(\Psi_B/2) \approx \Psi_B/2$$

Therefore:

$$\Psi_B/2 = dx_B/h_B = dx_B/L \quad (28)$$

Here, the length $U/2$ of the arc with radius r and angle $\Psi_B/2$ will be (see Fig. 3):

$$U/2 = r(\Psi_B/2) \quad (29)$$

Substituting Eqs. (27) and (28) in Eq. (29) yields:

$$\begin{aligned} U/2 &= [(U/2)^2 / (2\Delta Z_B)] (dx_B/L) \\ &= (U^2 dx_B) / (8L\Delta Z_B) \\ \therefore \Delta Z_B &= (1/4)(U/L) dx_B \\ &= [1/(4m)] dx_B \end{aligned} \quad (30)$$

Considering the apparent warp quantity $\Delta Z_A'$ with respect to Eq. (26):

$$\begin{aligned} \Delta Z_A' &= [1/(12m)] (\cos\theta) dx_A \\ \therefore \Delta Z_A' / dx_A &= [1/(12m)] (\cos\theta) \end{aligned} \quad (31)$$

Fig. 14 and Fig. 15 show the displacement ratios $\Delta Z_A' / dx_A$ derived from Eq. (31) for the No. 2 port and No. 3 port when m and θ are varied.

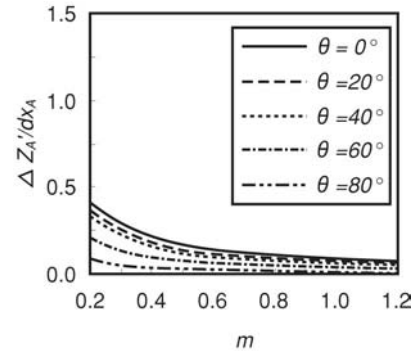


Fig. 14 Displacement ratio for No.2 and No.3 ports vs. aspect ratio "m". Displacement ratio increases as "m" and θ become smaller.

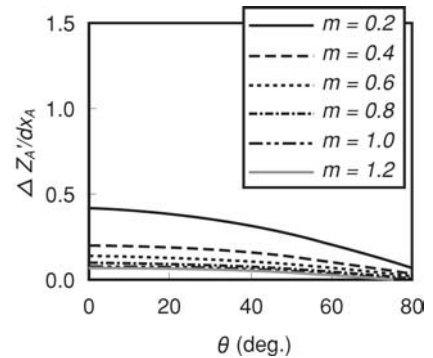


Fig. 15 Displacement ratio for No.2 and No.3 ports vs. deviation angle θ .

Similarly, from Eq. (30):

$$\Delta Z_B'/dx_B = [1/(4m)](\cos\theta) \quad (32)$$

Fig. 16 and Fig. 17 show the displacement ratios $\Delta Z_B'/dx_B$ derived from Eq. (32) for the No. 1 port and No. 4 port when m and θ are varied.

When θ remains the same, $\Delta Z_A'/dx_A$ and $\Delta Z_B'/dx_B$ diminish exponentially as the aspect ratio increases. When the aspect ratio remains the same, $\Delta Z_A'/dx_A$ and $\Delta Z_B'/dx_B$ increase as θ diminishes. When θ approaches 0° , however, this difference diminishes. The displacement ratio of No. 1 port and No. 4 port is several times larger than the displacement ratio of No. 2 port and No. 3 port.

2.5. The Branch Ratio with Respect to the Aspect Ratio

$$L/y = (L/U)(U/y) \quad (33)$$

Here, the offset ratio becomes $L/y = (y\cos\theta)/y$, so that, from Eq. (33):

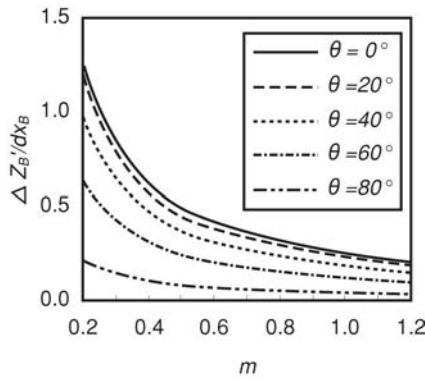


Fig. 16 Displacement ratio for No.1 and No.4 ports vs. aspect ratio “ m ”.
Displacement ratio increases as “ m ” and θ become smaller.
The displacement ratio for the No.1 and No.4 ports is fairly large compared with that for the No.2 and No.3 ports.

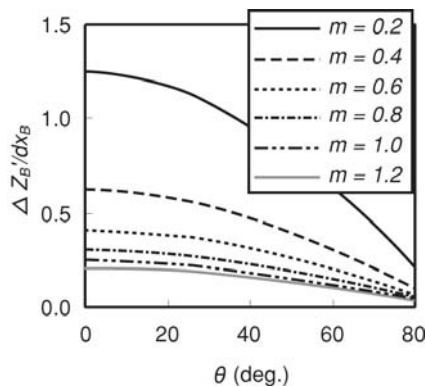


Fig. 17 Displacement ratio for No.1 and No.4 ports vs. deviation angle θ .

$$b = \cos\theta/m \quad (34)$$

The branch ratio derived from Eq. (34) when m and θ are varied is shown in Fig. 18.

When θ remains the same, the branch ratio diminishes nonlinearly as the aspect ratio increases. When the aspect ratio remains the same, the branch ratio is larger when θ is smaller.

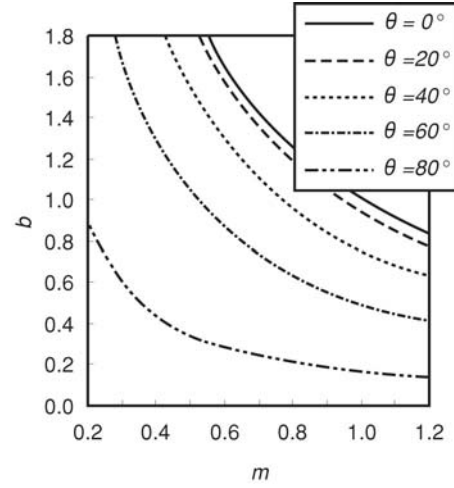


Fig. 18 Aspect ratio “ m ” vs. branch ratio “ b ”.

3. Discussion

The results obtained through the analysis can be summarized as follows. The stiffness of the exhaust manifold can generally be increased by making m smaller (Fig. 4 and Fig. 5), making θ smaller (Fig. 6 and Fig. 7), making b larger (Fig. 8 and Fig. 9), and making c larger (Fig. 12 and Fig. 13).

In order to increase the stiffness in this way, y should be made smaller, while it is better for U to be larger. However, the effect of L is reciprocal with respect to m and c . The effect on stiffness of θ is also the reverse of both m and b . For example, when U is determined by the engine dimensions, making L smaller will generally also make y smaller. In such a case, therefore, m becomes smaller when L is small, even though c does not vary to any great extent. Meanwhile, b becomes larger (even if U is fixed because y is small).

Similarly, the deformation of the exhaust manifold can be reduced by increasing both θ and m (Fig. 14 to Fig. 17).

Designing an exhaust manifold requires attention to various constraints imposed by the engine and the engine compartment. This requires the selection of dimensions and geometry that achieve an optimum balance among these parameters, which can be done by identifying the best trade-off among the parameters.

Next, on the assumption that θ , m , b , and c can be varied independently, their respective proportionate effects on non-dimensional stiffness are sought. This allows determination of the non-dimensional stiffness varying rate when θ , m , b , and c are varied individually. Consequently, it becomes possible to identify the respective proportionate effects on non-dimensional stiffness both more clearly and quantitatively when the above θ , m , b , and c are varied individually for different dimensions assigned to the various exhaust manifold parts.

Non-dimensional stiffness is a function of the above four independent variables θ , m , b , and c . Therefore, if:

$$Gx_A U^3/EI = f_A(\theta, m, b, c) \quad (35)$$

Then Eq. (35) yields:

$$\begin{aligned} d(Gx_A U^3/EI) &= (\partial f_A / \partial \theta) d\theta \\ &+ (\partial f_A / \partial m) dm \\ &+ (\partial f_A / \partial b) db \\ &+ (\partial f_A / \partial c) dc \end{aligned} \quad (36)$$

Here, Eq. (21) is applied so that:

$$\begin{aligned} \partial f_A / \partial \theta &= \partial [72b^3 / (b^2 \cos \theta + 24 \cos^3 \theta)] / \partial \theta \\ &= (72b^5 \sin \theta + 5184b^3 \cos^2 \theta \sin \theta) / \\ &\quad (b^2 \cos \theta + 24 \cos^3 \theta)^2 \\ &= T_{A\theta} \end{aligned} \quad (37)$$

Applying Eq. (19), then:

$$\begin{aligned} \partial f_A / \partial m &= \partial [72 / \{m[(1/\cos \theta) \\ &\quad + (24m^2/\cos^3 \theta)]\}] / \partial m \\ &= -72 \cos^3 \theta (\cos^2 \theta + 72m^2) / \\ &\quad (m \cos^2 \theta + 24m^3)^2 \\ &= T_{Am} \end{aligned} \quad (38)$$

Applying Eq. (21):

$$\begin{aligned} \partial f_A / \partial b &= \partial [72b^3 / (b^2 \cos \theta + 24 \cos^3 \theta)] / \partial b \\ &= (154b^4 + 5425b^2 \cos^2 \theta) / \\ &\quad (b^4 \cos \theta + 576 \cos^5 \theta + 48b^2 \cos^3 \theta) \\ &= T_{Ab} \end{aligned} \quad (39)$$

Applying Eq. (19):

$$\begin{aligned} \partial f_A / \partial c &= \partial [72 / \{m[(1/\cos \theta) \\ &\quad + (24m^2/\cos^3 \theta)]\}] / \partial c \\ &= (82c^4 + 5424m^2 c^2) / \\ &\quad (mc^4 + 576m^5 + 48m^3 c^2) \\ &= T_{Ac} \end{aligned} \quad (40)$$

Therefore, when Eqs. (37) through (40) are substituted in Eq. (36), the following Eq. (41) is obtained:

$$\begin{aligned} d(Gx_A U^3/EI) &= T_{A\theta} d\theta + T_{Am} dm \\ &+ T_{Ab} db + T_{Ac} dc \end{aligned} \quad (41)$$

Similarly:

$$Gx_B U^3/EI = f_B(\theta, m, b, c) \quad (42)$$

In this case, Eq. (42) yields:

$$\begin{aligned} d(Gx_B U^3/EI) &= (\partial f_B / \partial \theta) d\theta + (\partial f_B / \partial m) dm \\ &+ (\partial f_B / \partial b) db + (\partial f_B / \partial c) dc \end{aligned} \quad (43)$$

Eq. (22) is applied here so that:

$$\begin{aligned} \partial f_B / \partial \theta &= (180b^5 \sin \theta + 1296b^3 \cos^2 \theta \sin \theta) / \\ &\quad (5b^2 \cos \theta + 12 \cos^3 \theta)^2 \\ &= T_{B\theta} \end{aligned} \quad (44)$$

Applying Eq. (20):

$$\begin{aligned} \partial f_B / \partial m &= -36 \cos^3 \theta (5 \cos^2 \theta + 36m^2) / \\ &\quad (5m \cos^2 \theta + 12m^3)^2 \\ &= T_{Bm} \end{aligned} \quad (45)$$

Applying Eq. (22):

$$\begin{aligned} \partial f_B / \partial b &= (180b^4 + 1296b^2 \cos^2 \theta) / \\ &\quad (25b^4 \cos \theta + 144 \cos^5 \theta + 120b^2 \cos^3 \theta) \\ &= T_{Bb} \end{aligned} \quad (46)$$

Applying Eq. (20):

$$\begin{aligned} \partial f_B / \partial c &= (180c^4 + 1296m^2 c^2) / \\ &\quad (25mc^4 + 144m^5 + 120m^3 c^2) \\ &= T_{Bc} \end{aligned} \quad (47)$$

Therefore, by substituting Eqs. (44) through (47) in Eq. (43), the following Eq. (48) is obtained:

$$\begin{aligned} d(Gx_B U^3/EI) &= \\ &T_{B\theta} d\theta + T_{Bm} dm + T_{Bb} db + T_{Bc} dc \end{aligned} \quad (48)$$

The effects of θ and m on the displacement ratio is sought by a similar approach:

$$\Delta Z_A / dx_A = g_A(\theta, m) \quad (49)$$

Then, from Eq. (49):

$$\begin{aligned} d(\Delta Z_A / dx_A) &= (\partial g_A / \partial \theta) d\theta \\ &+ (\partial g_A / \partial m) dm \end{aligned} \quad (50)$$

Eq. (31) is then applied here:

$$\partial g_A / \partial \theta = S_{A\theta} = -\sin \theta / 12m \quad (51)$$

$$\partial g_A / \partial m = S_{Am} = -\cos \theta / 12m^2 \quad (52)$$

Substituting Eqs. (51) and (52) in Eq. (50), therefore, the following Eq. (53) is obtained:

$$d(\Delta Z_A'/dx_A) = (-\sin\theta/12m)d\theta - (\cos\theta/12m^2)dm \quad (53)$$

Similarly:

$$\Delta Z_B'/dx_B = g_B(\theta, m) \quad (54)$$

Therefore, from Eq. (54):

$$d(\Delta Z_B'/dx_B) = (\partial g_B/\partial\theta)d\theta + (\partial g_B/\partial m)dm \quad (55)$$

Here Eq. (32) is applied:

$$\partial g_B/\partial\theta = S_{B\theta} = -\sin\theta/4m \quad (56)$$

$$\partial g_B/\partial m = S_{Bm} = -\cos\theta/4m^2 \quad (57)$$

Consequently, by substituting Eqs. (56) and (57) in Eq. (55), the following Eq. (58) is obtained:

$$d(\Delta Z_B'/dx_B) = (-\sin\theta/4m)d\theta - (\cos\theta/4m^2)dm \quad (58)$$

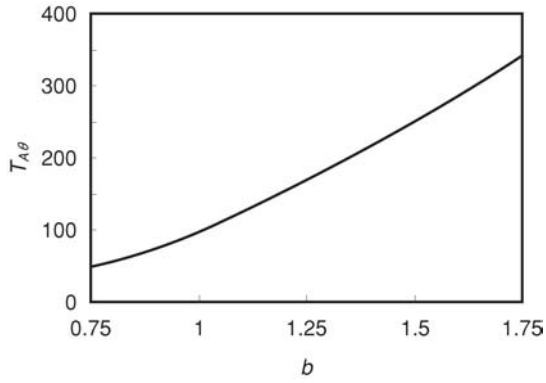


Fig. 19 “b” vs. “ $T_{A\theta}$ ” in the case of $\theta=60\text{deg.}$ (constant) derived from Eq.(37).

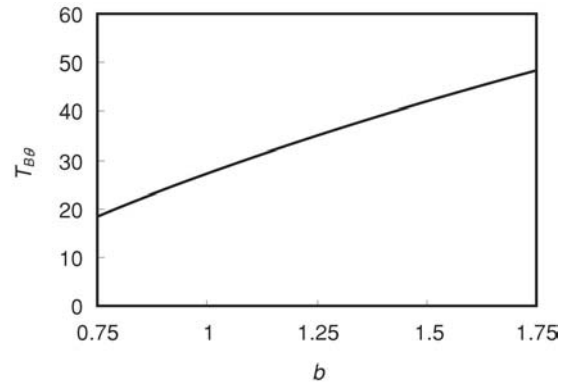


Fig. 22 “b” vs. “ $T_{B\theta}$ ” in the case of $\theta=60\text{deg.}$ (constant) derived from Eq.(44).

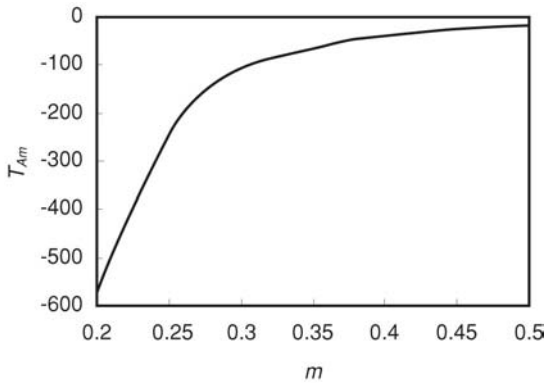


Fig. 20 “m” vs. “ T_{Am} ” in the case of $\theta=60\text{deg.}$ (constant) derived from Eq.(38).

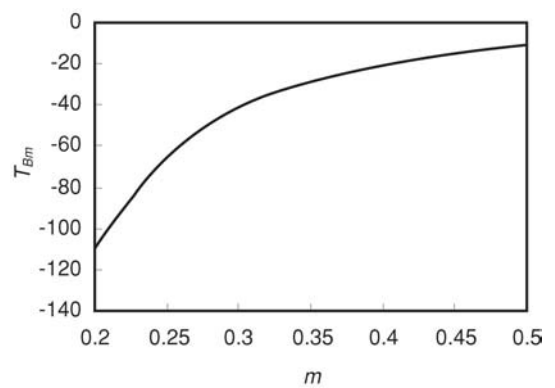


Fig. 23 “m” vs. “ T_{Bm} ” in the case of $\theta=60\text{deg.}$ (constant) derived from Eq.(45).

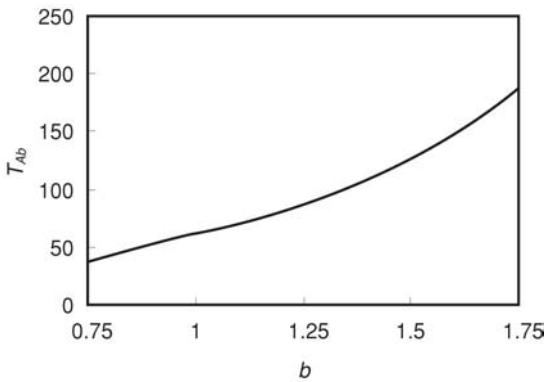


Fig. 21 “b” vs. “ T_{Ab} ” in the case of $\theta=60\text{deg.}$ (constant) derived from Eq.(39).

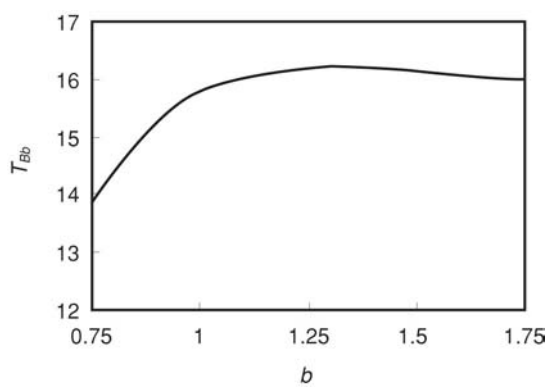


Fig. 24 “b” vs. “ T_{Bb} ” in the case of $\theta=60\text{deg.}$ (constant) derived from Eq.(46).

The above results make it possible to use Eqs. (37) to (40) and Eqs. (44) to (47) to obtain the several varying rate of stiffness $T_{A\theta}$, T_{Am} , T_{Ab} , T_{Ac} , $T_{B\theta}$, T_{Bm} , T_{Bb} , and T_{Bc} , which result from individual variations in θ , m , b , and c , respectively. The results in the case of $\theta = 60^\circ$ (constant) are shown in Fig. 19 through Fig. 24. By this method, the effects on stiffness varying rate resulting from individual variations in θ , m , b , and c , respectively, can be identified clearly and quantitatively.

These figures indicate, for example, that under the same conditions, $T_{A\theta} > T_{B\theta}$, $|T_{Am}| > |T_{Bm}|$, and $T_{Ab} > T_{Bb}$. It is apparent that the varying rate for No. 2 port and No. 3 port is greater than that for No. 1 port and No. 4 port. Also, the absolute values of T_{Am} and T_{Bm} are smaller when m is larger for both paired No. 2 and No. 3 ports and No. 1 and No. 4 ports. Similarly, T_{Ab} and T_{Bb} are smaller when b is smaller. In addition, the absolute values of T_{Am} and T_{Bm} are smaller when θ is smaller, as are T_{Ab} , T_{Bb} , $T_{A\theta}$, and $T_{B\theta}$ likewise.

Next, the varying rates $S_{A\theta}$, S_{Am} , $S_{B\theta}$, and S_{Bm} for the displacement ratio of each cylinder occurring when θ and m are varied are sought from Eqs. (51) and (52) in the case of No. 2 port and No. 3 port, and from Eqs. (56) and (57) in the case of No. 1 port and No. 4 port. The respective results are shown in Fig. 25 and Fig. 26.

These figures indicate that the absolute values of $S_{A\theta}$, S_{Am} , $S_{B\theta}$, and S_{Bm} are greater when m is smaller. This shows, therefore, that the several varying rates with respect to changes in m become larger when m is smaller. Furthermore, the absolute values of $S_{A\theta}$ and $S_{B\theta}$ increase as θ increases, but the absolute values of S_{Am} and S_{Bm} conversely grow smaller as θ increases. This clearly shows, therefore, that the several varying rates with respect to changes in θ are small.

Now the relationships identified above will be used to demonstrate a calculation of the quantitative effects on the varying rate of the stiffness and the varying rate of the displacement ratio occurring due to variations in θ , m , b , and c . Typical dimensions of an exhaust manifold for use with an in-line 4-cylinder engine will be used as an example.

The following values will be used:

$$\theta = 60^\circ$$

$$m = 0.35$$

$$b = 1.25$$

$$c = 0.5$$

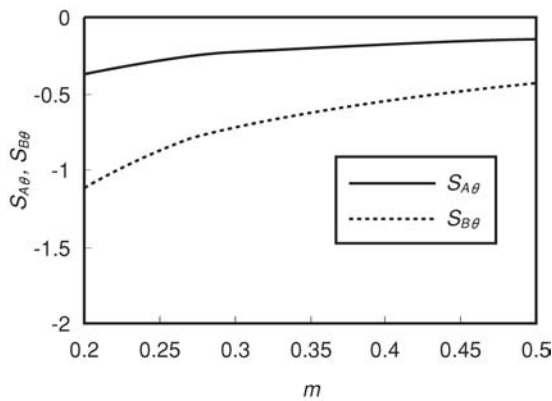


Fig. 25 “ m ” vs. “ $S_{A\theta}$ ” and “ $S_{B\theta}$ ” in the case of $\theta=60^\circ$ (constant) derived from Eq.(51) and Eq.(56).

Substituting the above values in Eqs. (37) through (40), the following results are obtained for No. 2 port and No. 3 port:

$$T_{A\theta} = 166.7$$

$$T_{Am} = -65.5$$

$$T_{Ab} = 87.3$$

$$T_{Ac} = 48.1$$

By similarly substituting values in Eqs. (44) through (47), the following results are obtained for No. 1 port and No. 4 port:

$$T_{B\theta} = 35.0$$

$$T_{Bm} = -28.1$$

$$T_{Bb} = 16.2$$

$$T_{Bc} = 19.7$$

This shows that when the variables for No. 2 port, No. 3 port, No. 1 port, and No. 4 port are varied the same amount, the factor that has the greatest effect on the stiffness varying rate is θ . This tendency is particularly marked in relation to No. 2 port and No. 3 port. The next largest effect is of m on No. 1 port and No. 4 port. Here, however, neither No. 2 port and No. 3 port, nor No. 1 port and No. 4 port, show any great difference between those effects and the effects of other factors b and c .

Eq. (51) and Eq. (52) yield the following warp quantities for No. 2 port and No. 3 port:

$$S_{A\theta} = -(\sin 60^\circ)/(12 \times 0.35) = -0.21 \quad (\text{for } \theta)$$

$$S_{Am} = -(\cos 60^\circ)/(12 \times 0.35^2) = -0.34 \quad (\text{for } m)$$

Similarly, Eq. (56) and Eq. (57) yield the following for No. 1 port and No. 4 port:

$$S_{B\theta} = -(\sin 60^\circ)/(4 \times 0.35) = -0.62 \quad (\text{for } \theta)$$

$$S_{Bm} = -(\cos 60^\circ)/(4 \times 0.35^2) = -1.02 \quad (\text{for } m)$$

For No. 2 port and No. 3 port, and No. 1 port and No. 4 port alike, the varying rate for m has a significant effect on the varying rate of the displacement ratio. The absolute values for both θ and m , which express the magnitude of that effect, are three times larger for No. 1 port and No. 4 port than they are for No. 2 port and No. 3 port.

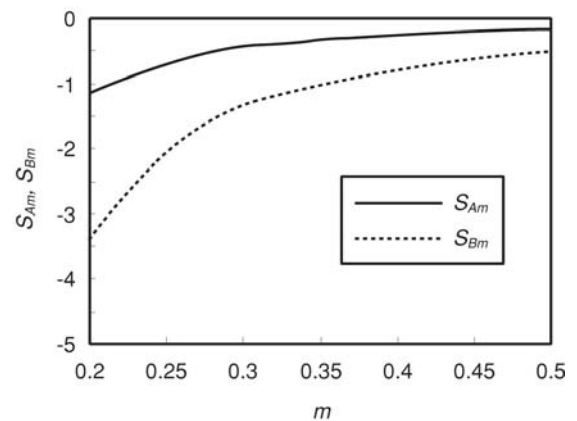


Fig. 26 “ m ” vs. “ S_{Am} ” and “ S_{Bm} ” in the case of $\theta=60^\circ$ (constant) derived from Eq.(52) and Eq.(57).

A method of the kind described above can be used to determine the respective quantitative effects exerted on the varying rates of the exhaust manifold stiffness and of the displacement ratio by θ , m , b , and c when these factors are varied independently. In other words, it becomes possible to know in advance the values of the stiffness and displacement ratio for the dimensions and shapes of the various parts of an exhaust manifold while it is being designed, as well as the quantitative changes that take place in the varying rate of the stiffness and displacement ratio when the dimensions of various parts are changed. The approaches and methods described here, therefore, can provide effective guidance for deciding the approximate dimensions of the various parts when a new exhaust manifold is being designed.

(October, 1993)

4. Conclusion

This research examined a desktop method for determining the stiffness of exhaust manifolds for in-line 4-cylinder engines and the displacement ratio that obtains during the application of load and moment to the port areas of those manifolds. In conjunction with this, the project also undertook to determine the effects on the varying rates of the stiffness and the displacement ratio that result when the dimensions and shapes of the various parts are varied. The method described here treats the exhaust manifold as a structure of beams in combination, thus allowing it to identify what effects the difference angle of the collective part, the branch ratio, the offset ratio, the aspect ratio, and so on have on the stiffness and the displacement ratio.

As a result, it was possible to identify the quantitative relationship of the dimensions and shapes of the various parts of an exhaust manifold for an in-line 4-cylinder engine to the stiffness and the displacement ratio, and also to clarify the quantitative effects of those dimensions and shapes on the varying rate of the stiffness and the displacement ratio. This is a method that can be used when newly designing an exhaust manifold for an in-line 4-cylinder engine as a handy and effective technique for deciding the approximate dimensions of the various manifold parts.

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