

Constructing n-sided G1 Patches with the Energy Integral Minimization

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ABSTRACT

A technique to create n-sided patches from n closed boundary curves with G1 continuity to the bounding surfaces has been developed. By using the data points on the boundary curves and the normal vectors of the bounding surfaces as the constraints, least squares patches are constructed with G1 continuity based on squares error minimization. Furthermore, the energy integral of the B-spline basis functions is calculated and the least squares patches are smoothed with energy integral minimization. Two coefficients, β (stretching term) and γ (bending term), are introduced as the elements used to adjust the smoothing. Consequently, the quality of the n-sided patches is improved. By using this technique the quality of the n-sided patches required for high quality styling design is improved and the time used to keep G1 continuity to the bounding surfaces is reduced.

1. Introduction

The problem of interpolating n-sided patches other than 4-sided patches is one of the central problems for CAGD (Computer Aided Geometric Design), and this is also one of the necessary functions for the development of CAD (Computer Aided Design) applications. Particularly when bounding surfaces exist, interpolating n-sided patches with G1 continuity to their bounding surfaces is important for creating high quality design data.

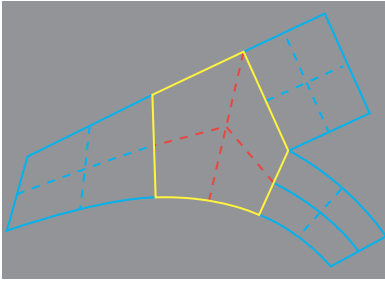
There are two typical approaches for interpolating n-sided patches. In the first method, an n-gon is divided into several quadrangles and then 4-sided patches are created for each quadrangle so that these 4-sided patches have G1 continuity with each other^{(1), (2)}. The key in this method is how to determine the internal curves used to divide the n-gon as the quality of the n-sided patches is influenced by these internal curves. However, there is no simple method to determine satisfactory internal curves, which causes problems in many cases. Fig. 1, Fig. 2 and Fig. 3 show the wire frame and zebra displays of 5-sided and 6-sided patches created with commercially available CAD

application using this method. It should be noted that the n-sided patches in these three cases are obviously unusable as styling surfaces.

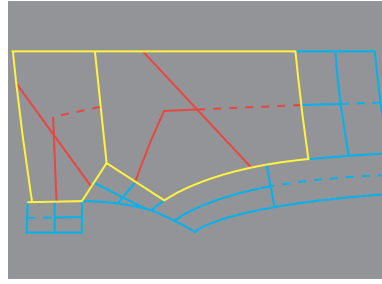
In the second approach, the least squares method is used to interpolate n-sided patches⁽³⁾⁻⁽⁵⁾. In this method, the data points are acquired from the n-gon boundary curves and a base surface that covers the n-gon region is created by solving the least squares equations. The n-sided patch to be interpolated is then completed by trimming the base surface along these boundary curves. Compared to the first approach, this method provides us with a single, distinct solution by directly solving equations and thus the arbitrariness involved in determining the internal curves is removed. This paper adopts the least squares method.

However, the quality of n-sided patches created using the least squares method depends on the data points, that is, the boundary curves, so in many cases surface smoothing must be considered. There are many methods developed for surface smoothing⁽⁶⁾⁻⁽¹⁰⁾. In this paper surface smoothing is achieved by calculating and minimizing the energy integral of the surface. Consequently, the quality of the n-sided patches is greatly improved.

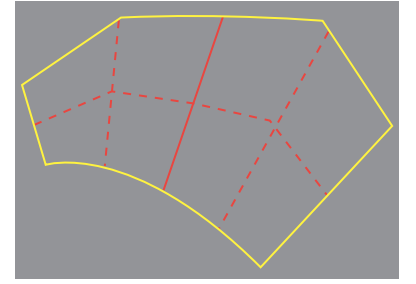
* Wako R&D Center



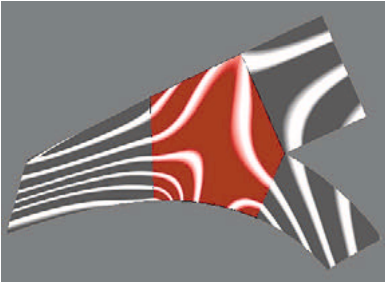
1-1 face



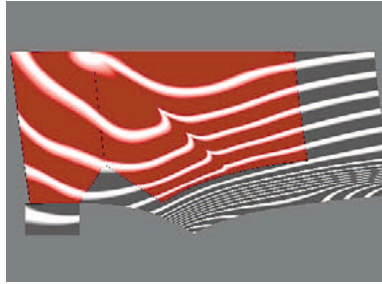
2-1 face



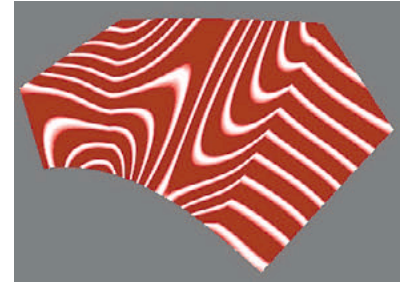
3-1 face



1-2 zebra



2-2 zebra



3-2 zebra

Fig. 1 Case 1: A 5-sided patch filled by CATIA

Fig. 2 Case 2: Two 5-sided patches filled by CATIA

Fig. 3 Case 3: A 6-sided patch filled by CATIA

2. Interpolation of the n-sided Patches by the Least Squares Method

2.1. Acquiring Data Points and Normal Vectors

When n closed boundary curves $curve[k]$ are given, the data points $\bar{Q}_a$ for applying the least squares method can be obtained from the boundary curves. If $len[k]$ is the length of curves respectively, l_0 is the maximum value of the lengths $len[k]$ and m_0 is the number of points taken from the longest curve, then the number of points $m[k]$ taken from each curve and the total number of points N_p are as follows:

$$N_p = \sum m[k] = \sum m_0 \cdot len[k] / l_0 \quad (1)$$

Here, m_0 should be determined according to the tolerance and calculation efficiency. (In this paper m_0 is set as 200. Thus, for a 5-sided patch the total number of points is 1,000 or less and for a 6-sided patch it is 1,200 or less). The data points $\bar{Q}_a$ ($0 \leq a < N_p$) taken from the boundary curves in this way should lie on the base surface $\bar{S}(u, v)$ that will be created by the data points, and they should satisfy the following equations.

$$\bar{S}(u_a, v_a) = \sum_{i=0}^{p_1-1} \sum_{j=0}^{p_2-1} N_i(u_a) M_j(v_a) \bar{P}_{ij} = \bar{Q}_a \quad (2)$$

Here, $N_i(u_a)$ and $M_j(v_a)$ are the B-spline basis functions in the parameter u and v directions, respectively. p_1 and p_2 , which represent the number of control points in the u and v directions of the base surface, can be defined as the sum of degrees and number of spans in the u and v directions, respectively. Further, $\bar{P}_{ij}$ are the unknown control points of

the base surface $\bar{S}(u, v)$, and their total number is $q = p_1 \cdot p_2$.

When bounding surfaces do not exist, denoting the tangent vectors at the intersection O_k of boundary curves $curve[k]$ and $curve[k+1]$ as $\bar{V}_k$, $\bar{V}_{k+1}$, and if the normal vector of plane composed from $\bar{V}_k$, $\bar{V}_{k+1}$ is $\bar{n}_k = \bar{V}_k \times \bar{V}_{k+1}$, then, as constraints for equation (2), the normal vector at

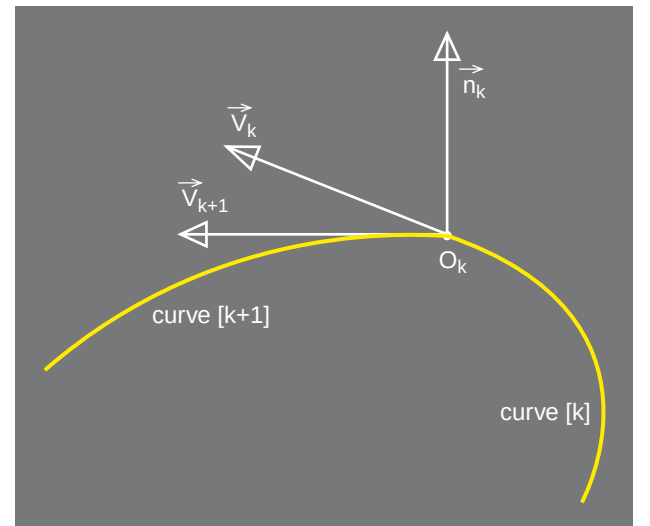


Fig. 4 A normal vector of the filling surface at the intersection of two boundary curves

the point O_k of the created n -sided patch must coincide with $\bar{n}_k$.

On the other hand, when bounding surfaces exist, the normal vectors $\bar{n}_a$ of the bounding surfaces at the related

data points $\bar{Q}_a$ lying on the boundary curves must be set as constraints for equation (2) in order to ensure G1 continuity between the created n-sided patches and the bounding surfaces. Assuming that the number of normal vectors $\bar{n}_a$ acquired from the bounding surfaces is N_n , the n-sided patches as well as the base surface satisfy the $2N_n$ constraint equations as follows:

$$\bar{n}_b \cdot \bar{S}_u(u_b, v_b) = \sum_{ij} N'_i(u_b) M_j(v_b) \bar{n}_b \cdot \bar{P}_{ij} = 0 \quad (3)$$

$$\bar{n}_b \cdot \bar{S}_v(u_b, v_b) = \sum_{ij} N_i(u_b) M'_j(v_b) \bar{n}_b \cdot \bar{P}_{ij} = 0 \quad (4)$$

Here, $\bar{S}_u(u, v)$ and $\bar{S}_v(u, v)$ represent the partial derivative of the base surface $\bar{S}(u, v)$ with respect to parameters u and v , respectively. $N'_i(u)$ is the derivative of $N_i(u)$, and $M'_j(v)$ is the derivative of $M_j(v)$.

2.2. Creating the Least Squares Plane

In order to calculate the B-spline basis functions and their derivatives in equation (2), (3) and (4), the location parameters (u_a, v_a) of the data points $\bar{Q}_a$ and the normal vectors $\bar{n}_a$ are required. The location parameters (u_a, v_a) for each point can be obtained from the point group $\{\bar{Q}_a\}$ as follows. First, the center of gravity G_0 of the point group $\{\bar{Q}_a\}$ and its squares error are calculated. Then, the principal axis of inertia and the principal moment of inertia of the point group $\{\bar{Q}_a\}$ are obtained. If the principal axis direction of the largest and second principal moments of inertia are $\bar{X}_p$ and $\bar{Y}_p$, respectively, then the vectors $\bar{X}_p$ and $\bar{Y}_p$ compose the least squares plane. Each data point $\bar{Q}_a$ is projected onto this least squares plane, and the projection coordinates on the coordinate system having the center of gravity G_0 as the origin are denoted as (x_a, y_a) . Subtracting the minimum projection coordinate $(x_{\min}, y_{\min})$ from (x_a, y_a) , the coordinate values $(x_a - x_{\min}, y_a - y_{\min})$ can be regarded as the location parameters (u_a, v_a) of the data points $\bar{Q}_a$. Fig. 5 shows an image of the creation of the least squares

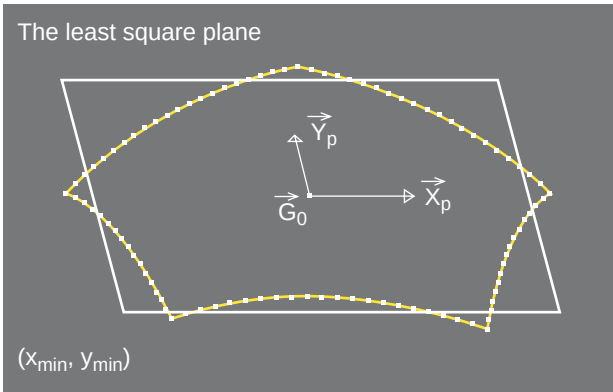


Fig.5 A least square plane obtained from a set of points on boundary curves

plane from the data points on boundary curves.

2.3. Creating n-sided Patches with G1 Continuity to the Bounding Surfaces

The B-spline basis functions $N_i(u_a)$ and $M_j(v_a)$ at points $\bar{Q}_a$ and their derivatives can be calculated using the parameters (u_a, v_a) obtained by projecting the data points $\bar{Q}_a$ onto the least squares plane. By rewriting vector

equations (2), (3) and (4) into coordinate component equations and formulating $3N_p + 2N_n$ simultaneous equations that include $3q$ unknowns x_i (coordinate components of unknown control points $\bar{P}_{ij}$), this can be expressed in matrix form as follows:

$$\begin{bmatrix} F \\ G \end{bmatrix} X = \begin{bmatrix} C \\ 0 \end{bmatrix} \quad (5)$$

Here, $F = [f_{ij}]_{3N_p \times 3q}$ is the coefficient matrix of the coordinate components in equation (2) and $G = [g_{ij}]_{2N_n \times 3q}$ is the coefficient matrix of the coordinate components in equations (3) and (4), and their elements are defined as follows:

$$\begin{aligned} f_{3a, 3(j \cdot p_1 + i)} &= N'_i(u_a) M_j(v_a) \\ f_{3a+1, 3(j \cdot p_1 + i)+1} &= N_i(u_a) M'_j(v_a) \\ f_{3a+2, 3(j \cdot p_1 + i)+2} &= N_i(u_a) M_j(v_a) \\ \text{others} &= 0 \end{aligned}$$

and

$$\begin{aligned} g_{2b, 3(j \cdot p_1 + i)} &= N'_i(u_b) M_j(v_b) \cdot n_{bx} \\ g_{2b, 3(j \cdot p_1 + i)+1} &= N'_i(u_b) M'_j(v_b) \cdot n_{by} \\ g_{2b, 3(j \cdot p_1 + i)+2} &= N'_i(u_b) M_j(v_b) \cdot n_{bz} \\ g_{2b+1, 3(j \cdot p_1 + i)} &= N_i(u_b) M'_j(v_b) \cdot n_{bx} \\ g_{2b+1, 3(j \cdot p_1 + i)+1} &= N_i(u_b) M_j(v_b) \cdot n_{by} \\ g_{2b+1, 3(j \cdot p_1 + i)+2} &= N_i(u_b) M_j(v_b) \cdot n_{bz} \end{aligned}$$

Furthermore, the matrices $X = [x_i]_{3q}$ and $C = [c_i]_{3N_p}$ of the coordinate components of the unknown control points $\bar{P}_{ij}$ and the given data points $\bar{Q}_a$ are respectively defined as follows:

$$\begin{aligned} x_{3(j \cdot p_1 + i)} &= P_{ij, x} \\ x_{3(j \cdot p_1 + i)+1} &= P_{ij, y} \\ x_{3(j \cdot p_1 + i)+2} &= P_{ij, z} \\ c_{3a} &= Q_{ax}, \quad c_{3a+1} = Q_{ay}, \quad c_{3a+2} = Q_{az} \end{aligned}$$

In the case of surface fitting, usually the number $3N_p + 2N_n$ in equation (5) is much larger than the number $3q$ of the unknowns x_i , so the problem is over-specified and only some average solution can be obtained. Indeed, by approximately solving the equation (5), a fair surface with few undulations can be created.

Equation (5) is solved below using the least squares method. The squares error sum of the point group $\{\bar{Q}_a\}$ and the squares error sum of the constraints due to the normal vectors $\bar{n}_a$ are respectively written as follows:

$$\begin{aligned} E_p &= \sum_{a=0}^{N_p-1} |\bar{S}(u_a, v_a) - \bar{Q}_a|^2 \\ &= (FX - C)^T (FX - C) \end{aligned} \quad (6)$$

$$E_n = \sum_{b=0}^{N_s-1} \left(\left| \bar{n}_b \cdot \bar{S}_u(u_b, v_b) \right|^2 + \left| \bar{n}_b \cdot \bar{S}_v(u_b, v_b) \right|^2 \right) \quad (7)$$

$$= (GX)^t(GX)$$

Here, $(FX-C)^t$ and $(GX)^t$ express the transposed matrices of $(FX-C)$ and (GX) , respectively. By minimizing the sum of the squares error sums (6) and (7),

$$E_{p+n} = E_p + E_n \quad (8)$$

the least squares equation for the unknowns X can be obtained as follows:

$$(F^tF + G^tG)X = F^tC \quad (9)$$

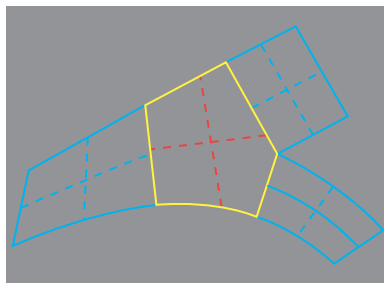
By using the solution of the equation (9), i.e. the control point coordinates of the surface, a base surface with G1 continuity to the bounding surfaces is created. Moreover,

if the base surface is trimmed at the boundary curve, then the n-sided patches, which interpolate the n-gon region, can be completed.

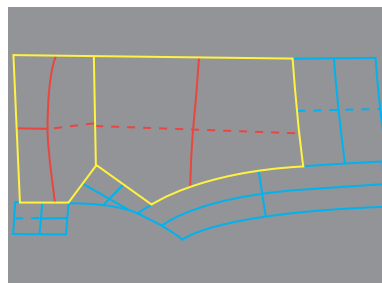
Next, the three cases shown in Fig. 1, Fig. 2 and Fig. 3 have been reconstructed by using the least squares method, and the following results have been obtained.

Comparing the results shown in Fig. 6 to Fig. 8 with Fig. 1 to Fig. 3, it is clear that the least squares fitting is superior for improving zebra continuity. Of the three cases, cases 1 and 3 have been greatly improved, and the 5-sided patch on the right side of case 2 has been also improved. However, there are also problems. Undesirable shapes such as spiral patterns are generated in the 5-sided patch on the left side of case 2.

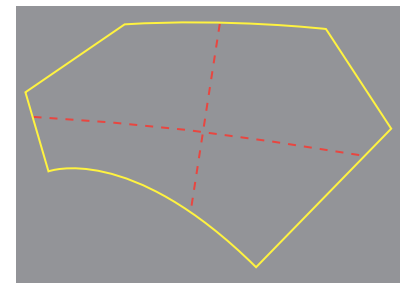
Using the least squares method, various cases of the n-sided patches have been examined, and the results show the following. (a) For a nearly flat n-sided patch such as shown in case 1, fine results can be obtained. (b) For other cases, the shortcomings that data points are not evenly distributed (they only exist on the boundary curves) becomes conspicuous, which causes two problems. The first is that the spiral patterns occur in the created n-sided patch as shown in case 2, and the second is that the base surface



1-3 face



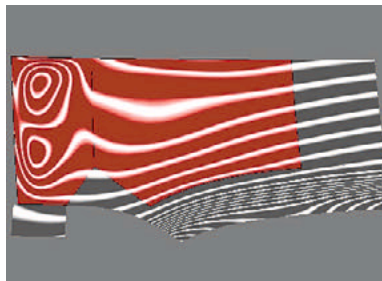
2-3 face



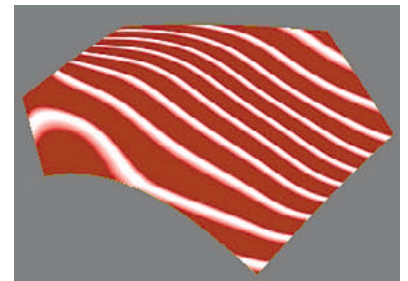
3-3 face



1-4 zebra



2-4 zebra



3-4 zebra

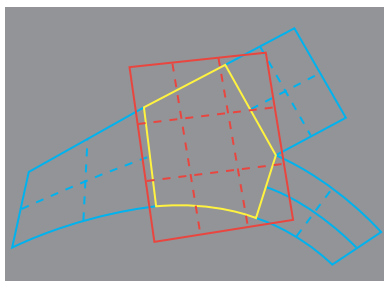
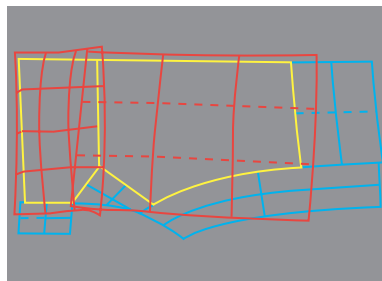
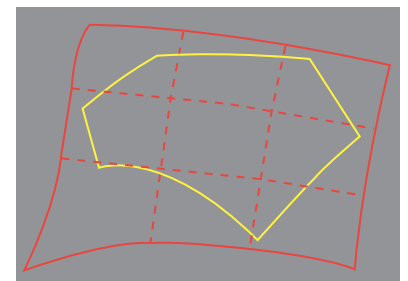
1-5 surface without trimming
maximum error = 0.00024mm2-5 surface without trimming
maximum error of the right
pentagon = 0.0017mm
maximum error of the left
pentagon = 0.00067mm3-5 surface without trimming
maximum error = 0.0016mm

Fig. 6 Case 1: A 5-sided patch filled by the least square method

Fig. 7 Case 2: Two 5-sided patches filled by the least square method

Fig. 8 Case 3: A 6-sided patch filled by the least square method

before trimming tend to diverge at the corners as shown in case 3. Satisfactory results cannot be obtained in most cases unless these two problems are solved. Therefore, surface smoothing by minimization of the surface energy integral will be considered as a solution to the above problems.

3. Smoothing by the Energy Integral Minimization

Surface smoothing is an important subject for CAGD, and various smoothing methods have been researched thus far. The main method is smoothing by minimization of the surface energy integral. In this paper, the square integrals of the first- and second-order derivatives of the B-spline basis functions with optional knot vectors are calculated. Surface smoothing is achieved by adding these integrals to the least squares equations and minimizing the energy integrals.

3.1. Square Integrals of the First-order derivatives

The square integrals of the first-order partial derivatives in the u and v directions of the surface $\bar{S}(u, v)$ are expressed as follows:

$$\begin{aligned} \iint |\bar{S}_u|^2 dudv &= \sum \int N_i' N_k' du \int M_j M_l dv \bar{P}_{ij} \cdot \bar{P}_{kl} \\ \iint |\bar{S}_v|^2 dudv &= \sum \int N_i N_k du \int M_j' M_l' dv \bar{P}_{ij} \cdot \bar{P}_{kl} \end{aligned}$$

If the subscripts are arranged as $a = j \cdot p_1 + i$ and $b = l \cdot p_1 + k$, and the integral portions for u and v are defined as follows:

$$\begin{aligned} Au_{ab} &= Au_{j \cdot p_1 + i, l \cdot p_1 + k} = \int N_i' N_k' du \int M_j M_l dv \\ Av_{ab} &= Av_{j \cdot p_1 + i, l \cdot p_1 + k} = \int N_i N_k du \int M_j' M_l' dv \end{aligned}$$

then the square integrals of the first-order partial derivatives are as follows:

$$\iint \left(|\bar{S}_u|^2 + |\bar{S}_v|^2 \right) dudv = \sum \bar{P}_a \cdot (Au_{ab} + Av_{ab}) \bar{P}_b$$

Furthermore, if the elements of the matrix $K_1 = [h_{ij}]_{3q \times 3q}$ are defined as follows:

$$\begin{aligned} h_{3a, 3b} &= h_{3a+1, 3b+1} = h_{3a+2, 3b+2} = Au_{ab} + Av_{ab} \\ others &= 0 \end{aligned}$$

the square integrals of the first-order partial derivatives that should be minimized can be expressed as follows:

$$E_a = \iint \left(|\bar{S}_u|^2 + |\bar{S}_v|^2 \right) dudv = X^T K_1 X \quad (10)$$

X in the above equation is the unknown control points, and the elements of the matrix K_1 can be calculated using the given knot vectors.

3.2. Square Integrals of the Second-order derivatives

The square integrals of the second-order partial derivatives in the u and v directions of the surface $\bar{S}(u, v)$ are expressed as follows:

$$\begin{aligned} \iint |\bar{S}_{uu}|^2 dudv &= \sum \int N_i'' N_k'' du \int M_j M_l dv \bar{P}_{ij} \cdot \bar{P}_{kl} \\ \iint |\bar{S}_{uv}|^2 dudv &= \sum \int N_i' N_k' du \int M_j' M_l' dv \bar{P}_{ij} \cdot \bar{P}_{kl} \\ \iint |\bar{S}_{vv}|^2 dudv &= \sum \int N_i N_k du \int M_j'' M_l'' dv \bar{P}_{ij} \cdot \bar{P}_{kl} \end{aligned}$$

If the subscripts are arranged in the same manner as in 3.1. and the integral portions for u and v are defined as follows:

$$\begin{aligned} Buu_{ab} &= Buu_{j \cdot p_1 + i, l \cdot p_1 + k} = \int N_i'' N_k'' du \int M_j M_l dv \\ Buv_{ab} &= Buv_{j \cdot p_1 + i, l \cdot p_1 + k} = \int N_i' N_k' du \int M_j' M_l' dv \\ Bvv_{ab} &= Bvv_{j \cdot p_1 + i, l \cdot p_1 + k} = \int N_i N_k du \int M_j'' M_l'' dv \end{aligned}$$

then the square integrals of the second-order partial derivatives are as follows:

$$\begin{aligned} \iint \left(|\bar{S}_{uu}|^2 + 2|\bar{S}_{uv}|^2 + |\bar{S}_{vv}|^2 \right) dudv \\ = \sum \bar{P}_a \cdot (Buu_{ab} + 2Buv_{ab} + Bvv_{ab}) \bar{P}_b \end{aligned}$$

Furthermore, if the elements of the matrix $K_2 = [k_{ij}]_{3q \times 3q}$ are defined as follows:

$$\begin{aligned} k_{3a, 3b} &= k_{3a+1, 3b+1} = k_{3a+2, 3b+2} \\ &= Buu_{ab} + 2Buv_{ab} + Bvv_{ab} \\ others &= 0 \end{aligned}$$

the square integrals of the second-order partial derivatives that should be minimized can be written as follows:

$$\begin{aligned} E_e &= \iint \left(|\bar{S}_{uu}|^2 + 2|\bar{S}_{uv}|^2 + |\bar{S}_{vv}|^2 \right) dudv \\ &= X^T K_2 X \end{aligned} \quad (11)$$

E_e on the left side of the above equation represents the surface energy, and K_2 on the right side is the stiffness matrix. The elements of the matrix K_2 can be calculated using the given knot vectors.

3.3. Surface Smoothing

In regard to smoothing, adding the first-order derivatives term E_a and the second-order derivatives term E_e to the equation of the squares error sum (8), the total term that should be minimized is then written as follows:

$$E_{p+n+a+e} = \alpha E_{p+n} + \beta E_a + \gamma E_e \quad (12)$$

Here, the subscripts p, n, a, e of $E_{p+n+a+e}$ mean point, normal, area and energy, respectively, and the non-negative fitting coefficients α, β and γ are introduced in order to enable adjustment of surface smoothing. Defining K_0 as follows:

$$K_0 = F^t F + G^t G$$

yields the new least squares equation.

$$(\alpha K_0 + \beta K_1 + \gamma K_2)X = \alpha F^t C \quad (13)$$

The first term on the left side of the above equation is the least squares method term derived from the data points and the normal vectors. The second term is derived from the first-order derivatives and the third term is derived from the second-order derivatives. Here, the second term is the stretching term for minimizing the surface area, and the third term is the bending term for minimizing the surface energy.

By giving appropriate values to the fitting coefficients α , β and γ and solving the equation (13), a smoothing base surface can be created as the underlying surface of the n-sided patches.

However, the important thing when solving the equation (13) is the relative values and not the absolute values of the coefficients α , β and γ . Therefore, it is sufficient to fix one of the three coefficients and then give appropriate

values to the remaining two coefficients. These coefficients are set as follows in our program in this paper.

$$\alpha = 1$$

$$\beta \text{ (key in) (Recommended value: } 0 \leq \beta \leq 0.0001)$$

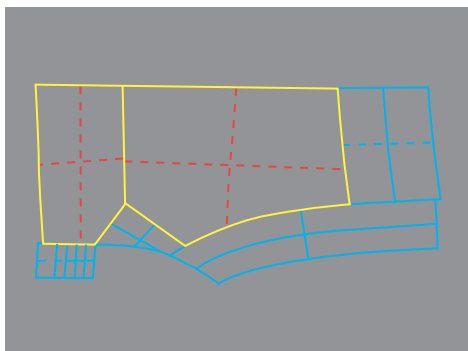
$$\gamma \text{ (key in) (Recommended value: } 0 \leq \gamma \leq 0.1)$$

Of course, if $\beta = \gamma = 0$ is input, the least squares equation (13) returns to the equation (9). However, the surface smoothing can be achieved by inputting $\beta \neq 0$ or $\gamma \neq 0$.

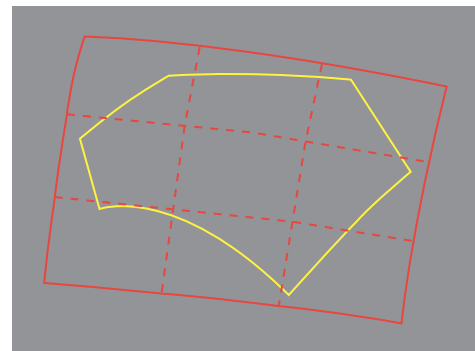
Next, we will discuss a method to improve the problems indicated in the section 2 above, i.e. the problem of the spiral patterns occurring and the problem of the divergence of the base surface at the corners.

Fig. 9 and Fig. 10 illustrate examples reconstructing cases 2 and 3 as shown in Fig. 7 and Fig. 8 by using the method of minimizing energy integrals. The maximum error, i.e. the maximum distance from the given boundary curves to the created n-sided patches slightly increases, while the quality of the n-sided patches is greatly improved. Hence the problems indicated in the section 2 are almost completely resolved.

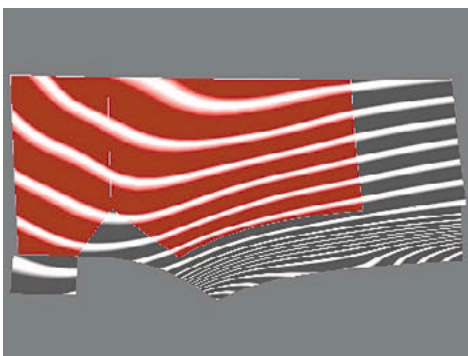
Let us look at the concrete effects of the stretching fitting coefficients β and the bending fitting coefficients γ in the equation (13) on the surface smoothing by examining



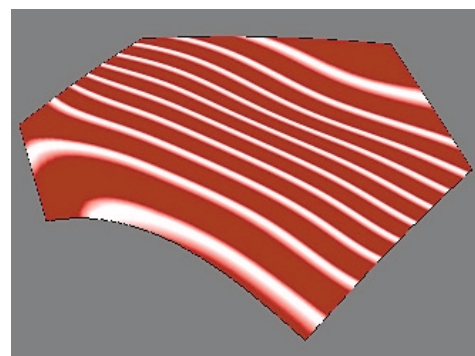
2-6 face



3-6 surface without trimming



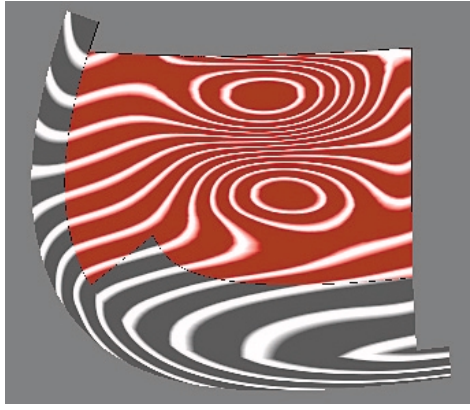
2-7 zebra
maximum error of the right pentagon
= 0.0032mm
maximum error of the left pentagon
= 0.00067mm



3-7 zebra
maximum error = 0.0023mm

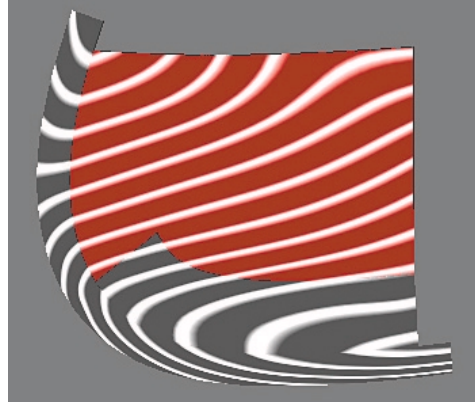
Fig. 9 Case 2: Two 5-sided patches filled by the minimal energy method with parameters $\beta=0$, $\gamma=0.01$

Fig. 10 Case 3: A 6-sided patch filled by the minimal energy method with parameters $\beta=0$, $\gamma=0.0001$



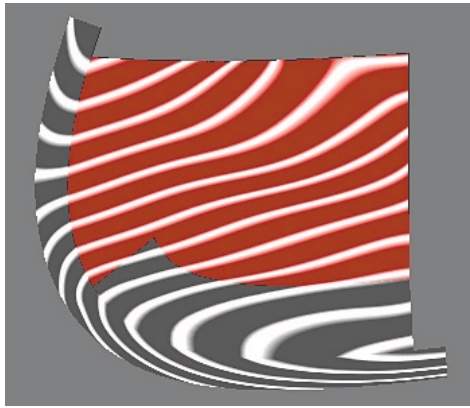
maximum error = 0.00087mm

Fig. 11 Case 4: A 5-sided concave patch filled by the minimal energy method with parameters $\beta=0$, $\gamma=0$



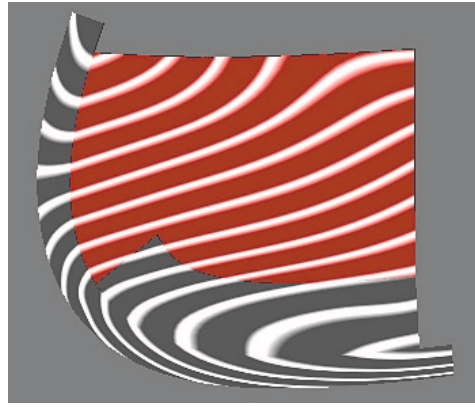
maximum error = 0.0012mm

Fig. 13 Case 4: A 5-sided concave patch filled by the minimal energy method with parameters $\beta=0$, $\gamma=0.01$



maximum error = 0.00099mm

Fig. 12 Case 4: A 5-sided concave patch filled by the minimal energy method with parameters $\beta=0.00001$, $\gamma=0$



maximum error = 0.0012mm

Fig. 14 Case 4: A 5-sided concave patch filled by the minimal energy method with parameters $\beta=0.00001$, $\gamma=0.01$

the case of a 5-sided concave patch.

The examples in Fig. 11 to Fig. 14 illustrate that the quality of the created 5-sided patch is gradually improved by adjusting the values of the fitting coefficients β and γ . As a result of studying various cases, it was concluded that β and γ value ranges of $0 \leq \beta \leq 0.0001$, $0 \leq \gamma \leq 0.1$ are appropriate.

4. Conclusion

As the fitting algorithm to create n-sided patches from n closed boundary curves with G1 continuity to the bounding surfaces, the method to interpolate a base surface to the boundary curves applying the least squares method and the surface smoothing method to minimize the energy integrals are presented. The quality of the n-sided patches in various cases has been then verified, and the following two conclusions have been reached.

- 1) The least squares fitting method has better results for the zebra continuity, improving upon the method of dividing

an n-gon into several quadrangles and then creating 4-sided patches for each quadrangle. However, problems such as unwanted spiral patterns and the divergence of the base surface at the corners are generated.

- 2) Surface smoothing by minimizing the energy integrals of the B-spline basis functions has been effective at improving surface quality. Attaching the fitting coefficients to the first-order and second-order derivatives terms in the least squares equation and adjusting the two coefficients β (stretching term) and γ (bending term), the problems of the spiral patterns and the corner divergence have been almost completely resolved.

By implementing this method in the Honda in-house CAD application as one of the functions to create surfaces, the quality of design data has been greatly improved. Concurrently, the time used to ensure G1 continuity between the n-sided patches and the bounding surfaces has been greatly reduced.

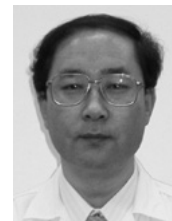
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