

Application of Helical Gear Tooth-root Bending Stress Simulation for Automotive Transmissions

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ABSTRACT

This technique was adapted from Professor Aizoh Kubo's program, which is capable of simulating tooth-root bending stress in helical gears, while taking into account the effects of misalignment and tooth surface configuration. A technique was created that is capable of more accurately simulating in a short time helical gear tooth-root bending stress in an actual automotive transmission during the first stages of design. The resulting tool was named GRANPLI. Misalignment in actual transmissions was measured using original gap sensors for a drive gear and driven gear behavior at low revolution speeds, and a misalignment map was created with respect to the loads. Particular attention was paid to the position of the primary reduction gear with respect to shaft orientation, the phase angle of the final reduction gear and the differential case, and so on. The appropriateness of calculation results obtained by GRANPLI was confirmed through comparison with data from measurements by strain gauges attached to a gear in an actual automotive transmission.

1. Introduction

There is an increasing demand for ever quieter and, at the same time, lighter and more compact automotive transmissions (referred to as transmissions hereafter) to improve fuel economy.

In addition, there is also call for the development of vehicles heavier than ordinary sedan passenger cars, exemplified by the sports utility vehicle (SUV), giving rise to higher loads on helical gears and other transmission components.

Inspections of the surfaces of helical gear teeth after testing the fatigue durability of a transmission under a constant load showed evidence in many cases of one-side contact in the meshed state. It is therefore necessary to have a gear strength prediction method for the final reduction gear, where the load is highest, that considers the effects of tooth surface configuration and misalignment.

Ever-shorter development periods also require a tool that can refine the design of helical gears from the first stages of the design process.

The authors carried out an analysis of the mechanism during helical gear meshing and proceeded with research on mesh fluctuation analysis of helical gears for automotive transmissions.

2. Conventional Helical Gear Strength Prediction

Conventional evaluation methods of tooth-root bending

stress in the helical gears of transmissions include the JSME, AGMA, BS, ISO, JGMA, DIN and other formulas. However, these formulas all share the basic concept for calculating bending strength⁽¹⁾.

When predicting the helical gear stress using these formulas, it is not possible to take misalignment or tooth surface configuration into account. Misalignment is the error in parallelism that occurs while the gear is in operation. It is a major factor contributing to bending stress in a transmission, because there are a wide variety of changes caused by a number of factors including the load transmission direction, the rigidity and backlash of the transmission case, bearings, gear shafts, and helical gears. Although it is therefore important to fully understand misalignment, it is very difficult to predict in the early stages of the design process when carrying out layout studies.

As a result, a lot of time is consumed in establishing and implementing many cases as criteria for judging the calculated results.

Although a method utilizing the finite element method (FEM) has recently been proposed to consider misalignment as an input condition, it requires a great deal of calculation time. Thus, it cannot be called the optimal method for use in the early stages of design.

Therefore, an alternative and more suitable method for predicting misalignment from the early stages of design was developed. This method applies a map for misalignment in regard to load under various conditions imposed by different layouts and component parts. This technique uses the program developed by Professor Aizoh Kubo of Kyoto

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University (referred to as the Kubo program hereafter). It allows misalignment to be considered when predicting the tooth-root bending stress. This new analytical tool is called "GRANPLI" (GeAR ANALyzing Program for Loading Influence).

3. Development of GRANPLI

3.1 Introduction of the Kubo Program

The Kubo program adopted in GRANPLI was modified in cooperation with Professor Kubo and his associates. This program can determine the various stresses generated in the gears according to the amount of rotational delay of drive gear to driven gear and allows for misalignment and tooth surface configuration. This allows the program to be applied to many different specifications for tooth height in both drive and driven helical gears and for meshing conditions that change in the tooth-width direction of helical gears used in automotive transmissions.

The Kubo program uses a function based on actual test results and the theories of Kubo, ⁽²⁾⁻⁽⁴⁾ Umezawa, ^{(2), (5)-(8)} and others for both tooth meshing and the tooth surface load distribution that accompanies it. The moment image method and the Aida and Terauchi formula ⁽⁹⁾ are also used in the bending stress calculations for the helical gear tooth root.

3.2 Program Modifications

The integral equation that expresses the load distribution on the single contact line of a helical gear and that forms the basis for this program is given in equation (1).

$$\int_0^{L(t)} K_b(x, \xi) p(\xi) d\xi + K_c(x) p(x) = \delta_0(x) = \{\Delta - e_{(x)}\} \cos \beta_g \quad (1)$$

- t : relative rotational angle on the gear
 $L(t)$: length of the contact line at location t
 x : coordinate of deflection measurement on the contact line
 ξ : coordinate of the point of application on the contact line
 $K_b(x, \xi)$: function of the effect of tooth shearing stress deflection for one pair of meshing teeth (referred to as the deflection effect function hereafter)
 $K_c(x)$: function of the effect of approach due to tooth contact for one pair of meshing teeth
 $p(\xi)$: distributed load
 $\delta_0(x)$: deflection of a pair of teeth at point x
 Δ : driven gear's delay angle in relation to the drive gear converted to a distance on the action line perpendicular to the shaft
 $e_{(x)}$: Compound error at point x of one pair of meshing teeth perpendicular to the shaft
 β_g : torsion angle on the base cylinder

The deflection effect function (K_b) is expressed by equation (2), ⁽⁶⁾ which is based on actual test results and a limit rack calculation analysis by the differential method according to Umezawa.

Deflection Effect Function:

$$K_b(x, y, \xi, \eta) = U \frac{v(r)}{\sqrt{F(x-\xi)} \sqrt{G(y-\eta)}} \sqrt{F(x)} \sqrt{F(\xi)} \sqrt{G(y)} \sqrt{G(\eta)} \quad (2)$$

$$\bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2}$$

- x : coordinate of the deflection measurement in the tooth widthwise direction
 ξ : coordinate of concentrated load in the tooth widthwise direction
 y : coordinate of deflection measurement in the tooth height-wise direction
 η : coordinate of the point of load application in the tooth height-wise direction
 U : absolute value of deflection
 $v(r)$: characteristic function of deflection
 $F(x)$: deflection characteristics at the point of load application in the tooth width-wise direction
 $G(x)$: deflection characteristics at the point of load application in the tooth height-wise direction
 $\bar{x}$: Values with a bar across the top are standardized for tooth height ($\bar{x} = x/h$, where h is the total tooth height)

Corrections have been made to both the values and the formulas with the Kubo program. The corrections were made so that the formulas and values match the various gear damage examples for various machines. The authors have limited themselves to relatively narrow tooth-width gears for automotive applications and have further revised the coefficients according to FEM analysis and tests of the following items.

- (1) The compensation coefficient for a low pressure-angle gear of 20° or less used in transmissions and not verified by the measurements carried out by Professor Umezawa and associates⁽⁶⁾.
- (2) The coefficient expressing the rigidity of the tooth tips in narrow tooth-width helical gears.

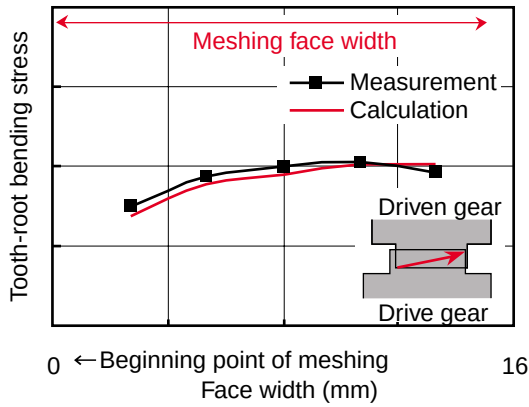
A verification of these coefficients was carried out by measuring the tooth-root bending stress while varying the gear meshing width. A high-rigidity gear unit test that makes it possible to ignore misalignment was used.

The specifications of the gears used in these tests are given in Table 1. Fig. 1 gives the measured and calculated results for the drive gear with a layout having a gear meshing face width of (a) 15mm and one of (b) 11mm.

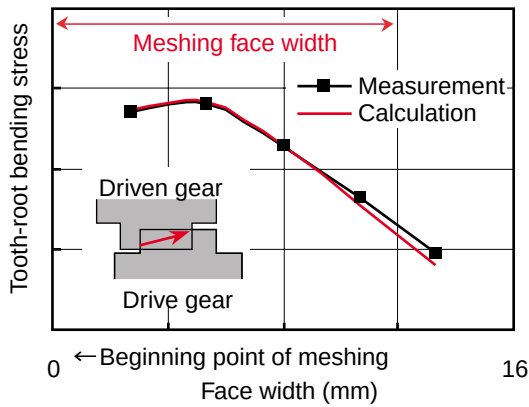
There is good agreement between the measured and calculated results, and it was thus confirmed that the set coefficient values were appropriate.

Table 1 Specification of gears used in experiments

	Drive	Driven
Normal module	2.00	
Standard pressure angle (°)	17.5	
Number of teeth	30	30
Face width (mm)	16.0	15.0
Helix angle (°)	36.0	



(a) Meshing face width: 15mm



(b) Meshing face width: 11mm

Fig.1 Calculation results of tooth-root bending stress distribution

3.3 Creation of the Misalignment Map

3.3.1 Measurement Conditions for the Misalignment in Transmissions

It is necessary to be able to predict misalignment in transmissions so that this program can be used effectively during transmission development.

This report presents a method for creating a misalignment map from measurements taken on a 5-speed manual Giacosa type transmission used in front-wheel-drive vehicles developed by Honda.

The reduction mechanism in this transmission comprises a helical gear and a conventional differential case as shown in Fig. 2. The structure of this differential case has open areas, so the rigidity is not constant throughout the case. This rigidity difference results in a variation in the misalignment of the final reduction gear according to its angular position.

In order to fully understand this misalignment, the behavior of the final reduction gear at a very low rotational speed was measured by using a test device that absorbs the load with a torque controller as shown in Fig. 3.

The specifications of the gears used for this test are given in Table 2.

3.3.2 Misalignment Measurements

Misalignment measurements were taken using a method that measures gear displacement with eddy-

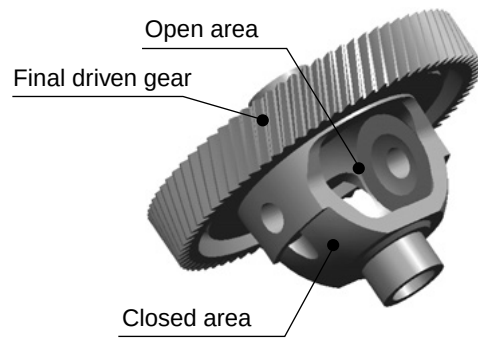


Fig. 2 Conventional differential case

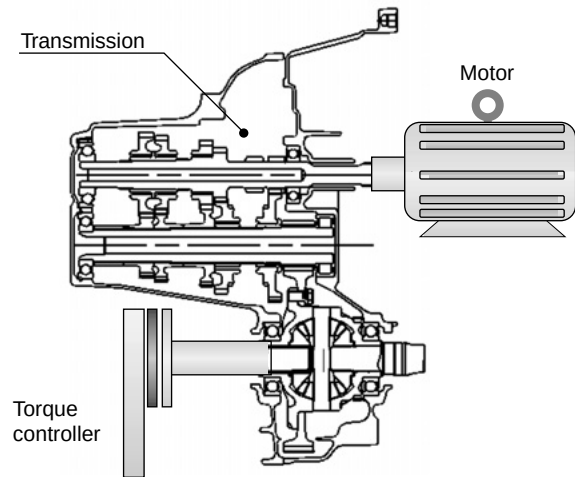


Fig. 3 Concept drawing of test equipment

Table 2 Specification of gears used in experiments

	Drive	Driven
Normal module	2.25	
Standard pressure angle (°)	17.5	
Number of teeth	18	79
Face width (mm)	37.5	33.0
Helix angle (°)	29.0	

current gap sensors.

Measurements were also taken so that the relative misalignment of the unit incorporating the differential case, bearings, shaft and gears could be evaluated.

The gap sensors and final reduction gear used in the gear test were positioned as shown in Fig. 4.

It was not possible with this arrangement to measure from the tip surface of the final reduction drive gear, so the behavior inside the shaft was measured.

The displacement was measured in two places – on the mesh lead side in the axial direction of the shaft, and on the mesh trail side. The relative cylindrical displacement is taken as the misalignment.

In order to understand the deflection components of the final reduction driven gear, measurements of the displacement on the gear tip surface were taken,

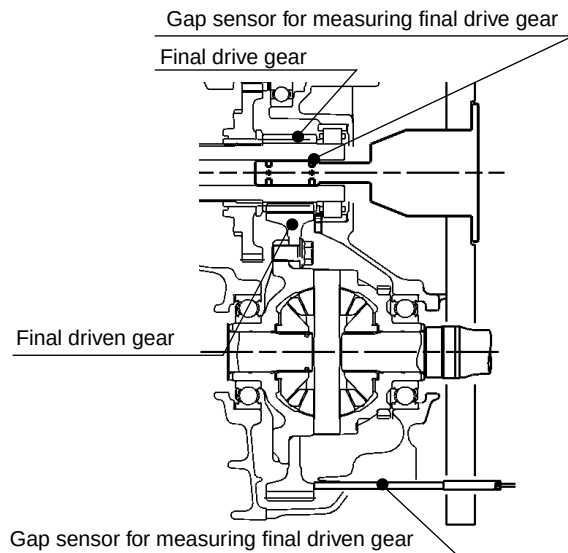


Fig. 4 Detail view of final gears

concentrating at a number of locations around the circumference and particularly near the mesh area.

These misalignment measurements enabled displacement data to be obtained for both the final reduction drive and driven gears, as shown in Fig. 5.

The spikes in the waveform for the final drive gear are marks that made it possible to identify the angular position of the differential case.

3.3.3 Misalignment Coordinate System

The misalignment coordinate system is based on Professor Kubo's definition shown in Fig. 6.

When the drive gear shaft moves off center with the driven gear shaft as the reference, the starting side shaft edge meshing surface directional component, when viewed from the meshing edge of the drive gear shaft, is AL. The component perpendicular to AL is AN. However, the direction away from the driven gear shaft is the positive direction.^{(10), (11)}

Let us now consider the effect that AL and AN have on meshing. When a point on a theoretical tooth surface reaches the plane of action due to the rotation of the gear, the distance from that point – according to the line of action – that the corresponding point on an actual tooth surface reaches on

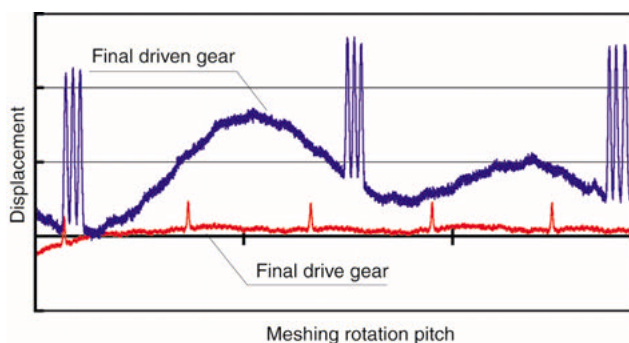


Fig. 5 Displacement by meshing rotation pitch

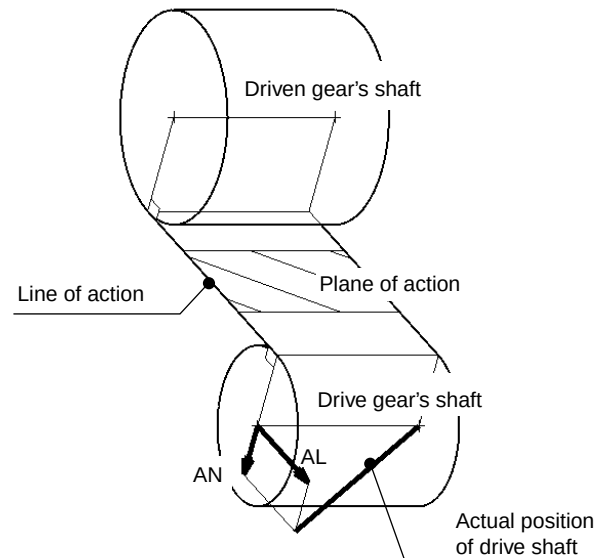


Fig. 6 Definition of misalignment

the plane of action determines the error value. If the vertical axis is set at this error value of said point of the plane of action, a single curved surface is created for the error of the entire surface. The curved surface determined from the plane of action is the error surface of the tooth shape. Calculation of the error surface according to the final reduction gear specifications given in Table 2 result in the error surfaces shown in Fig. 7. The effect of AL on the error surface was large, while the effect of AN on the error surface was small.⁽¹¹⁾ Fig. 7(a) is the curved error surface when AL is 50 μ m, and (b) is the error surface when AN is 50 μ m. However, these values were calculated for a tooth shape having no surface modifications.

Let us now consider the actual behavior of the meshing tooth surface. A change in the value of AL means that the meshing tooth surface is inclined in the tooth widthwise direction. This behavior promotes single-side contact. However, a change in the value of AN means that, although the meshing tooth surface is kept nearly parallel, this behavior reduces the meshing surface area. Considering this from the viewpoint of strength, it is considered that AN is a

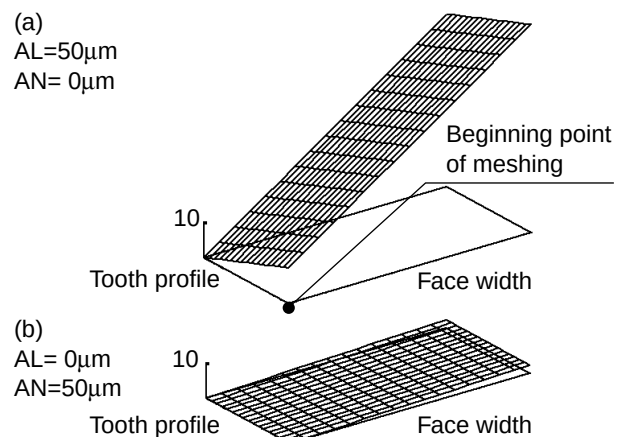


Fig. 7 Error surface

parameter that can be ignored to a reasonable degree in the early stages of design.

3.3.4 Misalignment of the Final Reduction Drive Gear

With the final reduction drive gear, the primary reduction gears and the final reduction gear are simultaneously in mesh. Consequently, differences in the load direction due to the proportion of each transmitted load, the support distance between the shaft bearings and each gear's location have different effects on gear displacement.

When measuring the final reduction drive gear displacement, tests were therefore focused on instances when the primary reduction gears were the 1st, 3rd and 5th gears. The specifications of the gears used in these tests are given in Table 3.

Fig. 8 shows the layout of the primary reduction gears and final reduction gear in the transmission.

Fig. 9 shows the measured and FEM-calculated results of behavior in the shaft when each primary reduction gear was engaged and transmitting power.

The results show the relative change in location of the mesh lead position when the mesh trail side is taken as the reference point of origin, in the tooth width. The measurements were taken with fixed conditions on the reduction drive gear for use in comparisons. Good agreement between the measured and FEM results confirmed the validity of the figures.

Finally, Fig. 10 shows the change in AL, which greatly affects the error surface, with respect to the torque when first gear was engaged and transmitting power.

Table 3 Specification of gears used in experiments

	1st gear	3rd gear	5th gear
Normal module	2.00	1.75	1.75
Standard pressure angle (°)	17.5	17.5	17.5
Gear ratio	3.266	1.212	0.738
Helix angle (°)	33.4	36.0	36.0

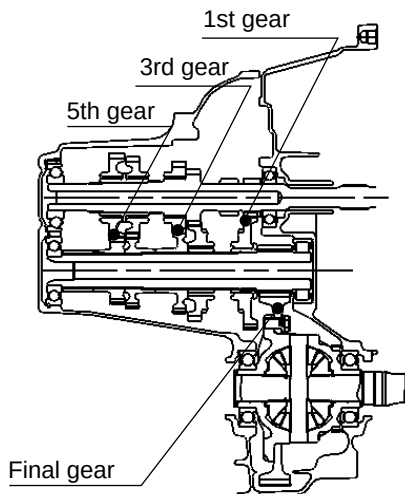


Fig. 8 Position of the primary reduction gears

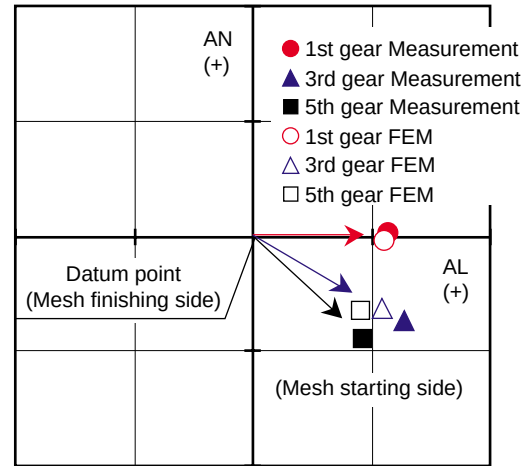


Fig. 9 Final drive gear position resulting in displacement

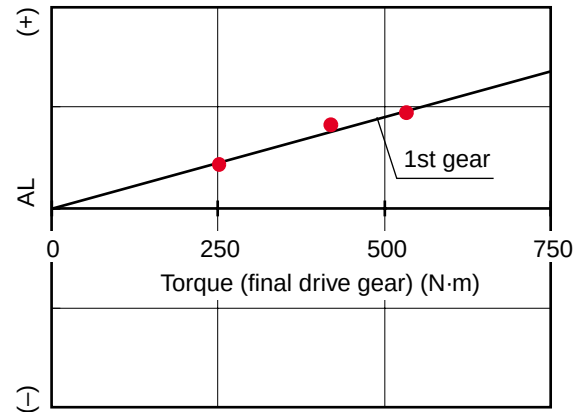


Fig. 10 Misalignment map (Final drive gear)

It can be seen from this graph that the misalignment changed in direct proportion to the torque over the range measured in these tests. This graph is applied as the misalignment map of the final reduction drive gear.

3.3.5 Final Reduction Driven Gear Misalignment Map

Fig. 11 shows the change in displacement of the final reduction driven gear tip surface with the changing structural rigidity of the differential case as the gear rotates. The origin represents the meshing location.

It can be seen from these plots that the steepest gradient was immediately before and after meshing. It was also confirmed that the gear tip surface displacement differed according to whether the open or closed portion of the differential case was meshing.

Fig. 12 shows the gear tip displacement as AL with respect to the rotational position of the differential case. The 0° angle occurs when the center of the open area in the differential case is adjacent to the point of meshing, with the positive angle in the direction of case rotation. As the case has a symmetrical structure, the range for the $\pm 90^\circ$ is shown.

It can be seen from Fig. 12 that the misalignment is so great that there is a change from the positive to the negative according to the rotational phase. Further, the maximum

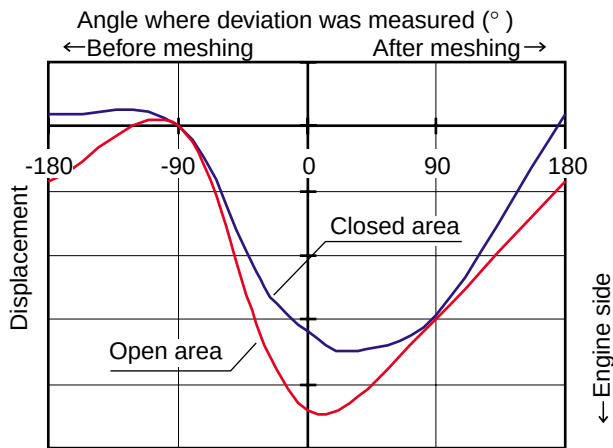


Fig. 11 Final driven gear displacement

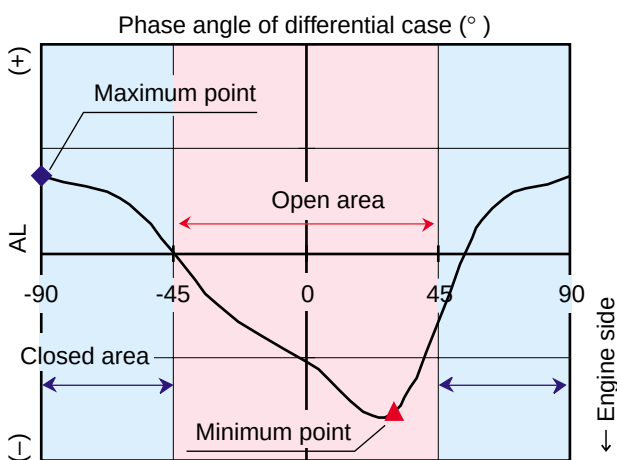


Fig. 12 Misalignment of the final driven gear according to the phase angle of the differential case

misalignment occurs when the closed area is adjacent to the point of meshing, and the minimum misalignment occurs when the open area is adjacent to the point of meshing.

Finally, Fig. 13 shows the change in AL with the torque, creating a misalignment map for final reduction driven gear.

It can be seen from Fig. 13 that even the misalignment in the final reduction driven gear changed in direct proportion to the transmitted torque in the range tested. The misalignment likewise varied according to the rotational angle of the differential case.

3.3.6 Relative Misalignment of the Final Reduction Gear

Combining the two misalignment maps given in Figs. 10 and 13 makes it possible to create a relative misalignment map for the final reduction gear as shown in Fig. 14.

4. Verification of the Validity of GRANPLI

4.1 Outline of the GRANPLI Verification Method

The validity of applying GRANPLI was verified by comparing calculated and measured values for the tooth-root bending stress. The stress measurements were taken at the same time as the misalignment measurements. The

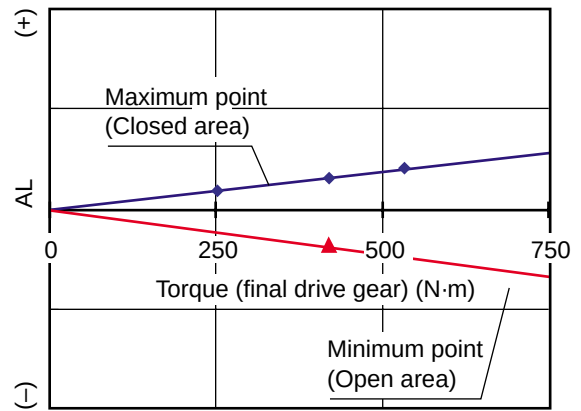


Fig. 13 Misalignment map (Final driven gear)

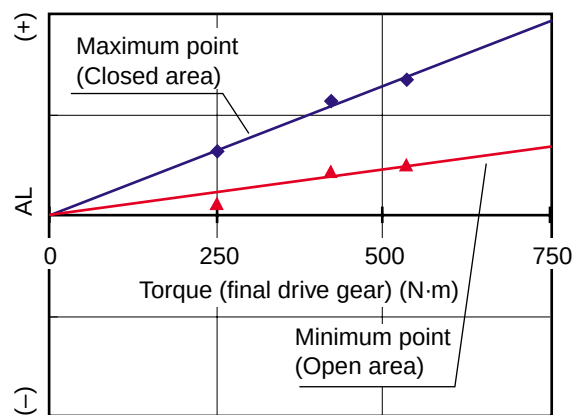


Fig. 14 Misalignment map (Final gear)

calculations by GRANPLI used the values for AL that were obtained from the misalignment map and the actual measured shape of the tooth surface.

4.2 Tooth-root Bending Stress Measurements

The tooth-root bending stress was measured on the pulled side of the tooth by strain gauges attached to the tooth root of the meshing tooth surface. Three strain gauges were attached at equal intervals as shown in Fig. 15 along the flank of a tooth at the location of the critical section in the meshing tooth surface tooth height direction as determined

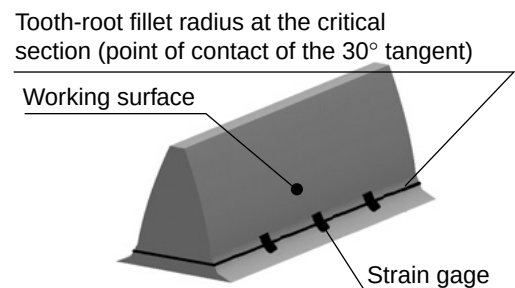


Fig. 15 Strain gage attachment points on final drive gear

by H. Hofer's 30° tangent method. The 0.2mm long gauges were set perpendicular to the tooth.

The measured data for the tooth-root bending stress is given in Fig. 16 along with the corresponding misalignment measurements recorded simultaneously.

It is possible to identify the phase angle of the differential case during the tooth-root bending stress measurements due to the relationship of the position of the wave forms along the rotational path.

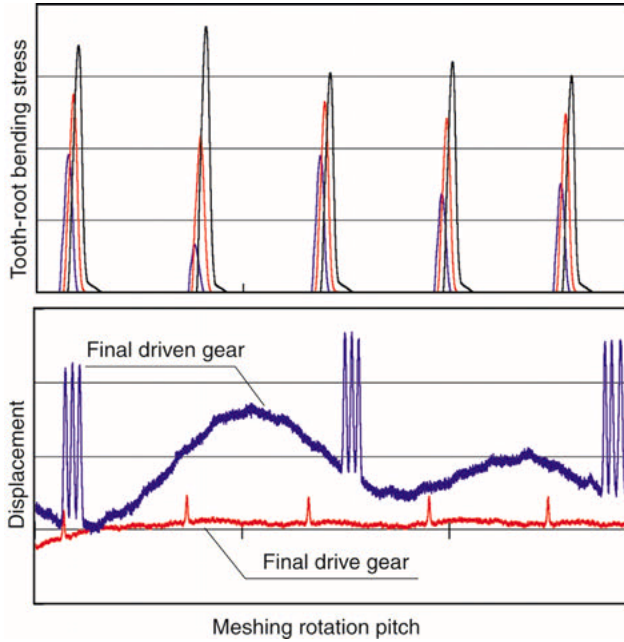


Fig. 16 Displacement and bending stress at tooth-root according to meshing rotation pitch

4.3 Calculation Conditions

The calculations for verification were carried out under the following conditions.

The calculations for misalignment applied the value at the torque obtained when measuring the tooth-root bending stress that was deduced from the misalignment map. The calculations were performed even when there was no misalignment for comparative purposes

As it is necessary to perform calculations for surface configurations under the same conditions as used for the measurements, the measured root surface configuration data shown in Fig. 17 was used: (a) final reduction drive gear data and (b) final reduction driven gear data.

4.4 Calculation Results

Fig. 18 shows the bending stress distribution on the drive side of the tooth root plotted against the tooth face width. Fig. 18(a) shows the greatest misalignment, while Fig. 18(b) shows the least misalignment.

Although calculations for the tooth-root bending stress show an equal distribution of the meshing range across the tooth face width when misalignment is not taken into consideration, the measured stress distribution is such that

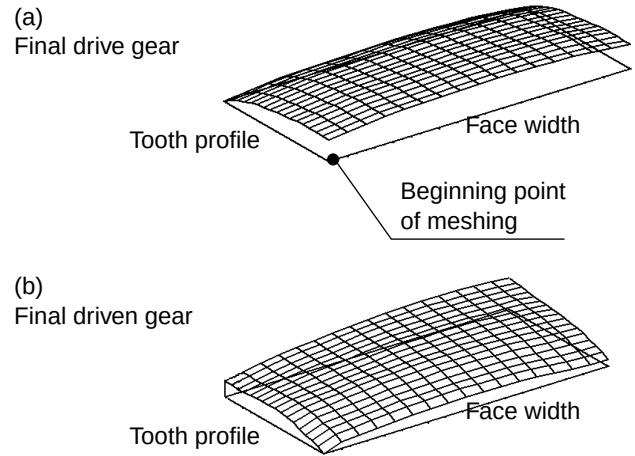
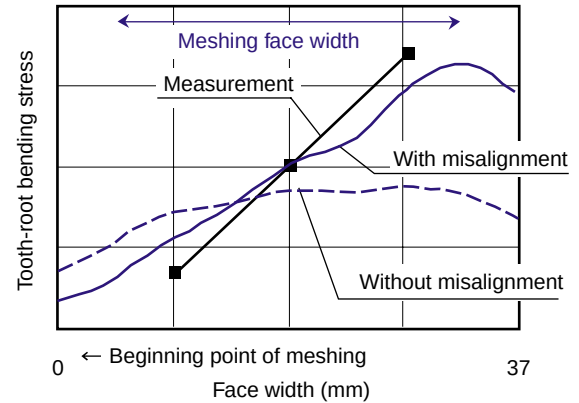
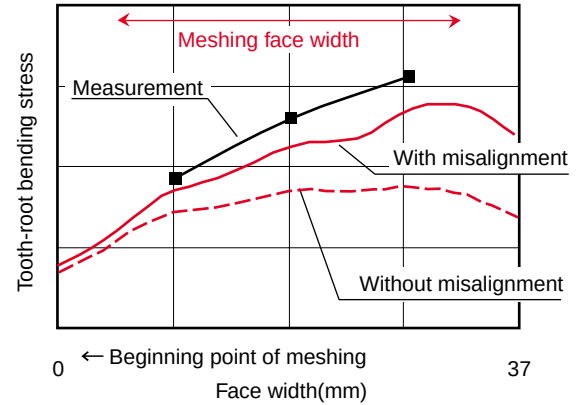


Fig. 17 Modified tooth flank of test gear



(a) Maximum point (Closed area)



(b) Minimum point (Open area)

Fig. 18 Calculation results of tooth-root bending stress distribution

the greatest tooth-root bending stress is generated on the mesh finish side. However when full consideration is given to the amount of misalignment obtained from the misalignment map, the calculated stress distribution also showed the greatest tooth-root bending stress is generated on the mesh finish side. Thus, results confirmed that accurate repeatability was possible.

The slope on both of the relevant misalignment maps is

reflected in the tooth-root bending stress distribution in respect to the open and closed areas of the differential case.

It was accordingly confirmed that the misalignment maps are appropriate for use.

4.5 Applications for Misalignment Maps

The applicability of the misalignment maps was next evaluated. The slope of the plot on the misalignment maps changed according to such conditions as the actual rigidity and backlash in the transmission. Creating a misalignment map makes it possible to apply these data to transmissions with different layouts.

The results were next verified by carrying out tests on the final reduction gear of a different transmission, the Giacosa type 4-speed automatic transmission. The final reduction driven gear of this Giacosa system is the same as that used in the preliminary study and has the same rigidity. Thus, a similar misalignment map could be used (Fig. 13). However, the final reduction drive gear is different in terms of the shaft layout and location of each primary reduction gear, so the misalignment map was modified to that shown in Fig. 19, with the relative misalignment map for the final reduction gear as shown in Fig. 20.

The GRANPLI calculation results from this misalignment map were compared with the tooth-root bending stress measurement results for the final reduction

driven gear. Fig. 21(a) shows the results when the misalignment was the greatest, and Fig. 21(b) shows them when it was the least.

It was confirmed that the calculated results for stress distribution by using the misalignment map could also be applied to transmissions with different layouts, since it was possible to accurately simulate the measured data.

5. Summary

The Kubo program, which allows for misalignment and tooth surface configuration in gear tooth-root bending stress prediction, was modified in conjunction with Prof. Kubo and associates so that the program could be applied in the early stages of automotive gear design. Misalignment has a large effect on the tooth-root bending stress. Thus, a map of misalignment in relation to the applied load was created by measuring gear behavior. This misalignment map makes it possible to more accurately predict gear tooth-root bending stresses in a short time in the early stages of design. This tool was named "GRANPLI."

The validity of the data calculated with GRANPLI was confirmed by comparing with measured tooth-root bending stress data.

The slope of the line on the misalignment map changes under a variety of conditions, but by using the most

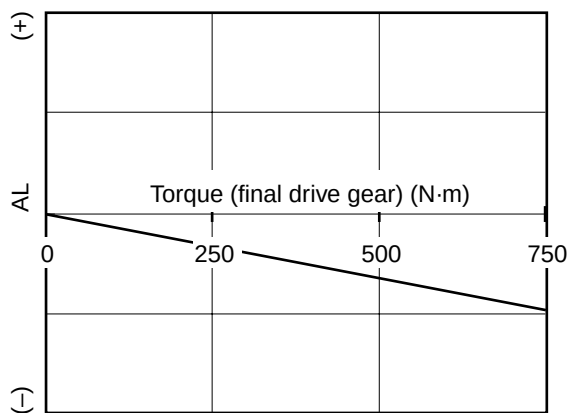


Fig. 19 Misalignment map (Final drive gear)

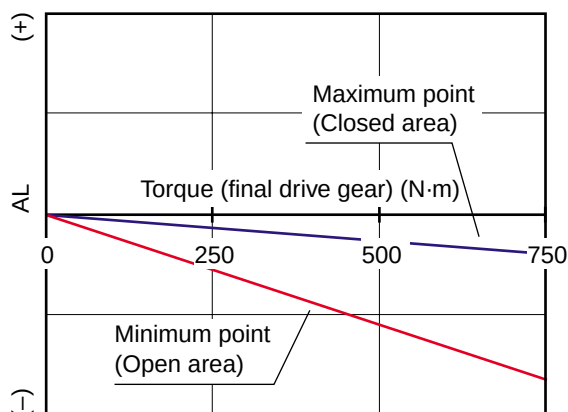
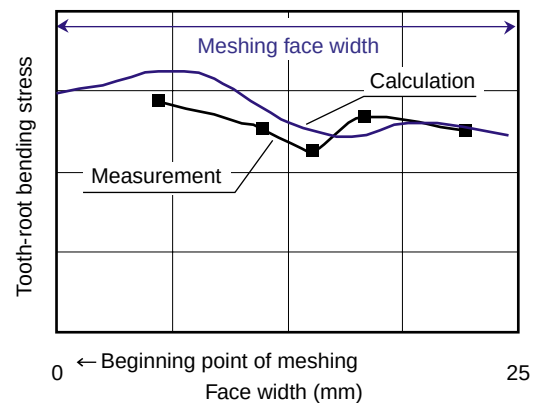
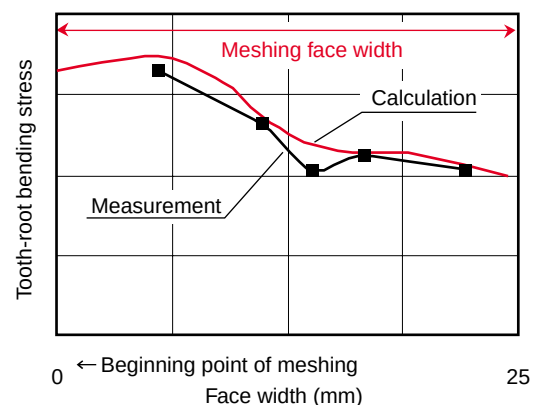


Fig. 20 Misalignment map (Final gear)



(a) Maximum point (Closed area)



(b) Minimum point (Open area)

Fig. 21 Calculation results of tooth-root bending stress distribution

appropriate misalignment map to suit the situation, this method can be applied to transmissions having different layouts.

This new method will be applied to create gear specifications in the early stages of the transmission design and to define early refinements to tooth surface configuration.

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References

- (1) Japan Society of Mechanical Engineers: Gear Strength Design Material, Tokyo, Japan Society of Mechanical Engineers, 1979, p. 205
- (2) A. Kubo, K. Umezawa: Research on Load Transmission Characteristics of Cylindrical Gears with Deviation (1st Report, Basic Principles), Transactions of the Japan Society of Mechanical Engineers (3rd Dept.) Vol. 43, No. 371, p. 2771 – 2783 (1977-7)
- (3) A. Kubo: Research on Load Transmission Characteristics of Cylindrical Gears with Deviation (2nd Report, The Effect of Gear Precision on Static Tooth-root Stress), Transactions of the Japan Society of Mechanical Engineers (3rd Dept.), Vol. 44, No. 378, p. 701 – 711 (1978-2)
- (4) A. Kubo, T. Ueno, J. Jin, Y. Ariura, T. Nakanishi: Basic Research into Gear Tooth Contact and Gear Performance (1st Report, Basic Principles), Transactions of the Japan Society of Mechanical Engineers (C edition), Vol. 51, No. 467, p. 1559 – 1567 (1985-7)
- (5) K. Umezawa, J. Ishikawa: Approach between Gear Teeth Due to Surface Contact, Transactions of the Japan Society of Mechanical Engineers (3rd Dept.) Vol. 38, No. 315, p. 2979 – 2987, (1972-11)
- (6) K. Umezawa: Meshing Test on Helical Gears under Load Transmission (1st Report, Approximation Formula for Deflection), Transactions of the Japan Society of Mechanical Engineers (3rd Dept.), Vol. 38, No. 308, p. 896 – 904 (1972-4)
- (7) K. Umezawa: Meshing Test on Helical Gears under Load Transmission (2nd Report, Approximation Formula for Bending-Moment Distribution), Transactions of the Japan Society of Mechanical Engineers (3rd Dept.), Vol. 38, No. 310, p. 1602 – 1608 (1972-6)
- (8) K. Umezawa: Meshing Test on Helical Gears under Load Transmission (3rd Report, The Static Behaviors of Driven Gear [sic]), Transactions of the Japan Society of Mechanical Engineers (3rd Dept.), Vol. 40, No. 330, p. 576 – 584 (1974-2)
- (9) T. Aida, Y. Terauchi, On the Bending Stress of Spur Gears, Transactions of the Japan Society of Mechanical Engineers (3rd Dept.) Vol. 27, No. 178, p. 853 – 876, (1961-6)
- (10) A. Kubo, T. Nonaka, T., Imamura: Equivalent Tooth Form Deviation and Vibrational Excitement due to Alignment Errors of Helical Gears, Japan Society of Mechanical Engineers Conference Papers, Vol. 68th, No. Pt. C, p.149 – 151, (1991)
- (11) T. Imamura: Changes in Combined Deviation Curves resulting from Gear Alignment Changes, and Their Effect on Gear Power Status, Kyoto University Eng. Dept., Precision Engineering Research Center Papers (1987-3)

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